Newton and Leibniz on the Relativity of Motion

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Abstract: In some respects there is more agreement between Newton and Leibniz on the relativity of motion than is generally recognized. Both were convinced Copernicans, and both accepted that motion could not be a merely extrinsic denomination, so that there would have to be true motions, and not just apparent ones. And both sought to discriminate true from apparent motions by reference to the causes of motion and to force. Where they differed was in their radically divergent understandings of how causes are related to forces, of how forces are to be determined in relation to quantity of motion, and of the ontological status of absolute or mathematical space. In this chapter I aim to throw light on the two philosophers' respective views on the relativity of motion by considering them in their historical genesis.

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Introduction

The basic point at issue between Leibniz and Newton on the relativity of motion can be stated simply enough. Leibniz held that "if several bodies are in motion, it cannot be inferred from the phenomena which of them is in absolute, determinate motion or at rest; rather, rest can be attributed to any of them you choose and the same phenomena will still be produced" (*Specimen dynamicum II*, [1695], GM VI 246-47).[1] Newton, on the contrary, held that absolute motions are distinguished from relative ones, and that in certain cases absolute motion *can* be inferred from the phenomena, in particular from "the forces of receding from the axis of circular motion" (Newton 1999, 412, 414). Here the *locus classicus* is his famous thought experiment with the rotating bucket, where the rise of the water up the bucket's sides "reveals its endeavour to recede from the axis of motion, and from such an endeavour one can find out and measure the true and absolute circular motion of the water" (413).

As a summary of who was right on the issue, and in what respects, the following, I believe, would pass as a statement of received wisdom among historians and philosophers of science. Newton was correct to perceive that the force involved in rotational motion serves as a criterion for the absoluteness (i.e. invariance under change of inertial frame) of that motion, but wrong to interpret this as a motion with respect to absolute space. Leibniz, on the other hand, was right to hold that the equivalence of hypotheses (i.e. the relativity of inertial motions, hereafter EH) makes it impossible to single out a preferred space, but wrong to think that this implies the universal relativity of motion. But Leibniz's appeal to the absoluteness of force in this connection is hard to understand: for if force is interpreted as mv^2, it is difficult to see how an appeal to it breaks the equivalence of hypotheses.[2] The impression that Leibniz was confused or inconsistent is then strengthened by his concession to Samuel Clarke in 1716 that the "absolute true motion of a body" can after all be distinguished from "a mere relative change of its situation with respect to another body".[3]

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[1] All translations in this article are my own.

[2] Daniel Garber (1995, 308-09) and John Roberts (2003, §4) have suggested that it implies the existence of a preferred frame of reference in Leibniz's system, and consequently an absolute speed in that frame.

[3] I will not address here related objections to Leibniz's relational theory of space regarding its alleged inability to support motion through time, and its assumption of the privileged nature of inertial motion in space. See Howard Stein (1977, 6-7) for criticisms, and Arthur (1994) for a partial defence of the viability of Leibnizian relationalism.

The obscurity of Leibniz's pronouncements, however, is at least partially caused by our viewing them in the light of post-Newtonian physics.[4] If instead we consider his and Newton's views as they were generated in their own historical context, some of the apparent confusions can be dispelled, and the motivations and assumptions underlying both their positions can appear more clearly. This is what I hope to achieve in this article, by examining how Newton and Leibniz developed their contending positions in a context that was determined both by their shared commitment to Copernicanism and by their differing criticisms of Cartesianism. From this perspective there is more agreement between them than one might have expected: both are firm advocates of Copernicanism, and both see the need to distinguish true motions from the merely apparent ones that would follow if all motion were relative, as Descartes had contended. Both, moreover, seek to do this by reference to the causes of the motions in question. This, however, is where their positions diverge. For their criticisms of Cartesianism lead them to radically incompatible understandings of how causes are related to forces, of how forces are to be measured, and of the ontological status of absolute or mathematical space.

Let me begin by outlining Leibniz's early writings on the relativity of motion, since these are much less widely known than they should be, and do much to illuminate his position.

1. The genesis of Leibniz's views on the relativity of motion

Leibniz's critique of Cartesianism was conditioned from the outset by two convictions: that all physical phenomena should be explained entirely mechanically – that is, in terms of the shape, size and motion of the constituent bodies; and that purely mechanical principles are insufficient to give an adequate grounding for physics, and need supplementing by metaphysical principles from which the laws of mechanics could be derived.

These convictions emerge in Leibniz's earliest efforts in physics, prompted by his discovery of the correction of Descartes's collision laws published by Wren, Hooke and Huygens in 1668-9. These authors pointed out that the quantity of motion conserved in collisions is not simply the sum of the products of the mass (or bulk, *mola*) m and speed v of the colliding bodies, but of the mass and the *speed in a given direction* (what we would call the vector velocity, $\underline{v}$), $m\underline{v}$. Leibniz accepted

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[4] Typical in the apparatus of commentators on Leibniz's and Newton's natural philosophy are such concepts as co-ordinate systems and universal reference frames, the distinction between kinematic (force-free) and dynamic, and the very notion of a spacetime structure. Understanding Newton's force in modern terms is also anachronistic, since it disqualifies inertia (mv) and impulse (Δmv) as forces, contrary to Newton's understanding.

this result, but argued that it had not been adequately grounded mechanically, since mass is not explicable in terms of size, shape and speed alone. But his own attempts to derive the laws of impact using only the concept of endeavour, published in his *Theory of Abstract Motion* early in 1672, were not a success, as he had already realized by the time he reached Paris that Fall.

Leibniz drew two important conclusions from this failure. The first was that since there is nothing in body as conceived in the mechanical philosophy to offer any resistance to motion, it is therefore necessary to posit some passive power in matter distinct from pure activity (with activity conceived here as endeavour) in order to account for the correct rules of collision. The second conclusion Leibniz drew from this may have been suggested to him by his conversations with Huygens while they were both in Paris (1672-76).[5] This concerned the fact that the rules of collision he had published in his *TMA*, like Descartes's own rules of collision, presupposed a generic extension with respect to which certain of the colliding bodies would be at rest and others moving with determinate velocities. Thus they were not invariant under a change of hypothesis as to which bodies could be taken as being at rest. But the empirically established rules of collision are invariant under a change of hypothesis: if a body of mass m_1 has velocities $\underline{u}_1$ before and $\underline{v}_1$ after the collision (with respect to some body or system of bodies taken to be at rest), and the other body of mass m_2 has velocities $\underline{u}_2$ before and $\underline{v}_2$ after the collision, then according to the conservation of (vector) quantity of motion, $m_1(\underline{u}_1 - \underline{v}_1) = - m_2(\underline{u}_2 - \underline{v}_2)$. Now, if these velocities are computed instead from the standpoint of a body moving at a velocity $\underline{w}$ with respect to the first, then the velocity differences $(\underline{u}_1 - \underline{v}_1)$ and $(\underline{u}_2 - \underline{v}_2)$ will remain the same, and the conservation of quantity of directed motion will be unaffected. The conservation law underpinning the correct laws of collision depends only on velocity differences, that is, on the relative motions of the bodies concerned. Leibniz recognized this, writing in an unpublished manuscript from April 1676 that "the conservation of the quantity of motion should be asserted of the action, i.e. relative motion, by which one body is referred to another, or acts on another" (A VI iii 493/LoC 77-79). If the laws of collision are premised on a space in which the motions are taking place, and are not invariant to a change of frame (like Descartes's and Leibniz's collision laws in his *TMA*), then it would be possible to identify absolute space as the space in which they hold. If, on the other hand, they appear the same in all

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[5] We know that Huygens gave Leibniz critical feedback on his *Theory of Abstract Motion* in Paris; and also that they discussed the relativity of motion together then, since in his letter to Huygens of 12 June 1694 Leibniz recalls that at that time Huygens was "of Mr. Newton's opinion with regard to circular motion", which Huygens confirms in his reply of August 24th. See Stein 1977, 40.

inertial frames of reference—as appears to be the case, and as they will be if motion is merely relative—then the supposition of an absolute space is rendered unnecessary. Leibniz made this point explicitly in a manuscript written shortly after his return to Hanover in early 1677:

> If *space* is a certain thing supposed in pure extension, whilst the nature of *matter* is to fill space, and *motion* is change of space, then motion will be something absolute; and so when two bodies are approaching one another, it will be possible to tell which of them is in motion and which at rest; or, if both are moving, with what speed they are moving. And from this will follow those conclusions which I once showed in the *Theory of motion abstractly considered*. But in reality space is not such a thing, and motion is not something absolute, but consists in relation. ("Space and motion are really relations", A VI iv 1968/LoC 225)

It is worth emphasizing that this rejection by Leibniz of absolute space as a privileged frame of reference for determining motions occurs well before he has any inkling of Newton's views, and a full ten years before Newton's first publication of them in his *Principia* (*Philosophiae naturalis principia mathematica*) in 1687. It is also worth remarking that the centrality of the collision laws is not arbitrary here: the mechanical philosophy insisted that all physical action is by direct bodily contact, i.e. collision. So Leibniz did not doubt that if the relativity of motion is implicit in the laws describing collisions, then it would apply to all physical actions.

Nevertheless, this is an a posteriori argument for the relativity of motion, and Leibniz was convinced that relativity is implicit in the very nature of motion, geometrically understood. Leibniz argues for this, giving perhaps his most thorough treatment of the relativity of motion, in another early manuscript, the *Principia mechanica* (*Mechanical Principles*), most probably penned in Paris in the summer of 1676.[6] The treatise is ambitious: Leibniz proposed to derive the principles of mechanics from geometry (more accurately, phoronomy, since figures are considered as movable). But once he gets onto the topic of motion, the issue of its relativity takes centre stage, and occupies him for the majority of the manuscript.

Since motion, geometrically understood, is simply change of situation (*situs*), he begins by defining the latter. "Situation," he writes, "is a mode according to which any body can be found". and this "depends on recognizing its distance from other bodies, and also on recognizing the angle, that is, the figure which it makes with another body" (A VI iii 103). This is essentially Hobbes's characterization of situation. Leibniz's attempt here at a more general definition marks an initial step

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[6] A VI iii 101-10; translation and commentary in Arthur 2013c.

toward his science of situation, the Analysis Situs, developed in numerous drafts from 1679 until his death in 1716, but never brought to a successful completion. In its dependence on distances and angles, it presupposes Euclidean space. But the idea is to characterize all the algebraic relations holding among situations, and then use them to define situations algebraically, jettisoning the Euclidean framework as a mere scaffolding. Situation reduces to the recognition of distance,[7] and "distance is the shortest path from one thing to another".[8]

With this characterization of situation in hand, Leibniz proceeds to argue that change of situation is entirely *respective*, i.e. dependent on the point of view from which the moving bodies are observed. First he considers four cases on the supposition that "there are only two bodies *A* and *B* in the world". The appearances will be the same whether *B* is at rest and *A* is moving towards it with a uniform velocity $\underline{v}$, or *A* is at rest and *B* moving towards it with a uniform velocity $-\underline{v}$, or *A* and *B* are moving along a line towards one another with velocities $\frac{1}{2}\underline{v}$ and $-\frac{1}{2}\underline{v}$, or *A* and *B* are moving uniformly in the same direction with a difference in velocities of *v*. Then he considers whether, by assuming an eye in a third body *C* (assumed to be at rest) observing the motions of *A* and *B*, "something certain can be determined concerning the absolute and proper speed of bodies." But again all the phenomena – all the mutual situations at each instant – will appear the same, and this is so even when *C* is allowed to move in the same direction as *B* but with half its velocity, as he considers in case 6. Therefore, Leibniz concludes, not even an omniscient being will be able to determine which body is in absolute motion: "whatever speed or direction we attribute by assuming an absolute motion for one of the bodies, we will always find that anyone must then understand motion in the others in such a way that everything will appear as before" (109).

This conclusion, of course, conflicts with our normal attributions of motion. "No one doubts," Leibniz notes, "that the stagecoach moves over the ground rather than the ground under the coach" (104-5). The reason for this is that we know the *cause of the motion* of the coach, and in such cases we are able to distinguish motion from mere change of situation. "In the case of two bodies, he writes, "motion is attributed to that one which contains the cause of their mutual situation having changed" (104). Thus when people "go for a walk, they believe they are approaching the town, rather than the town approaching them, because they feel some fatigue and

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[7] Here Leibniz moves too fast: situation depends on distance *and angle*. For example, in the case of a body rotating around its own centre, its distances from other bodies remains the same while its angle changes.

[8] Since situations involve distance, Leibniz's construction of space (assuming its success) would have avoided Clarke's accusation that "space and time are quantities, which situation and order are not" (*Third Reply*, §4; LC 32).

exertion in themselves" (104). The expenditure of work allows us to identify that it is we who are walking, or that it is the horses pulling the coach.

In more complicated scenarios, however, the cause of motion must be identified by reference to the *simplest hypothesis* for explaining the phenomena in question. Although "not even the least determination can be found for excluding any of the various possible hypotheses" about which particular bodies are in absolute motion (110), still "we will be permitted to choose the simpler mode of explaining, which involves reference to a cause from which the remaining changes may be derived more easily" (111). Thus in the case of a solid body moving through a liquid, "we will not hesitate to attribute motion to the solid body from which we can deduce the undulations of the surrounding liquid, rather than thinking of those undulations as originative" (111). Leibniz will continue to use this same example in his mature work to explicate his philosophy of cause. Thus in the *Specimen inventorum* of the late 1680s he writes that in the case of a solid moving through a liquid, the hypothesis of the solid's moving is "infinitely simpler than the others", and "that thing from whose state a reason for the changes is most readily provided is adjudged to be the cause" (A VI iv 1620/LoC 311). The changes of situation are mere *extrinsic denominations*, whereas the reason for them is intrinsic to the state of the thing causing the motion.

Significantly, this distinction between purely *geometrical motion* (change of situation) and *motion with respect to cause* is used by Leibniz in the *Mechanical Principles* to argue for the Copernican hypothesis. He notes many considerations in support of both the annual motion and the diurnal motion of the Earth, including the far greater simplicity of the Copernican hypothesis in dispensing with the imaginary epicycles and eccentric circles, the potential changes in the apparent diameters of the fixed stars and changes of situation of the Earth relative to the fixed stars, and observations of oscillations of hanging lamps and tides "impinging only on eastern and western shores" (A VI iii 105). As he concludes, "these things can be explained more distinctly by the supposed motion of the Earth and its being reduced to a simple cause" (111).

This does not mean, however, that we can conclude that the Earth is moving in an absolute sense. Leibniz acknowledges that a rotating body will be accompanied by centrifugal motion: "when a solid body is rotated around its own center it throws off contiguous bodies along the straight line that is tangent to the circle of the motion" (110). But, Leibniz objects, the same changes of relative situation are compatible with a much more complicated hypothesis, where the body is stationary and a fluid is moving around it with compensatory motions in such a way that all the same changes of relative motion are produced:

> it does not follow that it can be determined with absolute or mathematical certitude that the solid body is moving rather than at rest, since one may always imagine various compositions of motion in the parts of a liquid through which the same phenomena will be explained with the solid at rest; even if these suppositions are remarkably complicated, and that is the simplest which rather attributes motion to one solid and derives from it the motion in the parts of the liquid.

Thus, he concludes, "no certain knowledge can ever be had concerning absolute motion and rest from the phenomena of the changed situation alone" (110). He was even more explicit in notes he had written on Descartes's *Principia* at the end of 1675: "Throwing off along the tangent does not argue the real motion of the rotating thing, since it would be the same if everything moved around it" (A VI iii 215/LoC 25).[9] At any instant, a body on the surface of the rotating body will tend to move along a tangent with respect to the surrounding fluid; but their mutual situations and relative motion would remain the same at any instant if the body were stationary and the fluid were revolving around it. Granted, it would be an enormously complicated hypothesis that could account for all the motions in this way if the body were supposed to remain at rest through time with the fluid rotating around it. Leibniz took this to show (1) that if motion is understood purely geometrically, the universal relativity of motion must be upheld, and (2) that it is nevertheless possible to determine the subject of motion by reference to the causes of the motions, by choosing that hypothesis "from which the remaining changes may be derived most easily" (111). Thus whether the earth is moving or the fluid matter surrounding the earth is moving in complex ways, "these things can be explained more distinctly by the supposed motion of the earth and its being reduced to a simple cause" (111). Although Leibniz does not explicitly say so, this argument also counts in favour of the Copernican Hypothesis over the Tychonic.[10]

Leibniz maintained and elaborated on this position in his later writings. In a manuscript he drew up while in Rome in 1690, he advised his Italian colleagues that they "could freely follow Copernicus without damaging the authority of the censors, if only they were to recognize, with us,

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[9] The last phrase is a paraphrase of Seneca, from his discussion of Aristarchus, the "ancient Copernicus". Leibniz makes the same tacit classical allusion in a letter to Perrault in 1676, and in a manuscript written in Rome in 1690 (see LoC 383-84).

[10] Cf. Leibniz's comment on Descartes's discussion of relativity in 1675: "when things are mutually changing their situation and it is asked which of them should be said to be moving, motion should always be ascribed to a certain finite thing rather than to the rest of the world outside it." (A VI iii 217/LoC 29).

that the truth of a hypothesis should be taken as nothing but its greater intelligibility" (C 592). Thus when the subject of motion is determined according to the most intelligible hypothesis, one will be able to identify the true motions, such as the diurnal and annual motions of the earth. Similarly, in his *Dynamica* of 1695, Leibniz wrote: "Universally, when motion occurs, we find nothing in bodies by which it could be determined except change of situation, which always consists in relation (*in respectu*). Thus motion by its nature is respective. Meanwhile," though, "we attribute motion to bodies according to those hypotheses by which they are most aptly explained, and the truth of the hypothesis is nothing other than its aptness." (GM VI 507-508; Part II, §3, Prop 19). All this explains how Leibniz could grant Clarke that "there is a difference between an absolute true motion of a body and a mere relative change of its situation with respect to another body" (Fifth Paper to Clarke, §53/LC 74). For, he explains: "when the immediate cause of the change is in that body, that body is truly in motion".

Leibniz's use here of the word 'absolute', however, is misleading. For these true motions are not absolute motions in Newton's sense, motions with a determinate velocity in a supposed absolute space. They are motions relative to other bodies that are immobile according to the most intelligible hypothesis, and this hypothesis is relative to the phenomena to be explained. Thus, for example, the Sun (or a point near it) may be taken to be immobile if one is explaining the motion of the planets, but the whole solar system may itself be moving relative to the fixed stars. As Leibniz had written in February 1677,

> The absolute motion we feign to ourselves, on the other hand, is nothing but an affection of our soul while we regard ourselves or other things as immobile, since when these things are regarded as immobile we are more easily able to understand everything together. (A VI iv 1970/LoC 229)

This means that space itself is just an order of situations that we posit in order to represent these motions calculated according to the most intelligible hypothesis.[11] It is a preferred reference space for explaining these phenomena, but this does not give it the kind of ontological status supposed by Newton, since the phenomena might all look the same if that body were at rest relative to some other bodies taken as immobile. As Leibniz wrote in another fragment from the late 1680s (perhaps after he had read Newton's *Principia*):

[11] See Arthur 2013b for an account of Leibniz's complex views on space.

> Absolute space is no more a thing than time is, even though it is pleasing to the imagination; indeed, it can be demonstrated that such things are not real things, but merely relations of the mind trying to reduce everything to intelligible hypotheses, that is, to uniform motions and immobile places, and deducing values from this. (A VI iv 1638/LoC 333)

So for Leibniz the attribution of causes does not establish an absolute motion in Newton's sense. An appeal to the most intelligible hypothesis will determine which bodies are truly moving, and with what velocities. But these will not be velocities in absolute space, but relative to certain bodies taken as immobile on this hypothesis. This explains how Leibniz could insist that there is a true motion, associated with a most intelligible (and therefore true) hypothesis for explaining these phenomena, and yet still insist that "it cannot be inferred from the phenomena which of them is in absolute, determinate motion or at rest". There is a space associated with the most intelligible hypothesis for explaining each particular set of phenomena, but this will not constitute a unique space for explaining all phenomena. Thus for him the appeal to causes breaks the equivalence of hypotheses aetiologically, but not ontologically.

Consequently, Leibniz had an entirely different understanding from Newton of how causes of motion are related to forces. This he developed in the new science he called "dynamics", whose beginning can be dated to early 1678 with his composition of the manuscript *De corporum concorsu*. Building on his earlier insights about causes and the expenditure of work, Leibniz came to the realization that the correct measure of the force of a body's motion in the universe is not after all the quantity of motion in it, since observable motion can be converted into a force capable of producing an equivalent motion even when no motion is manifested – as in a pendulum at the top of its swing. He explicated this in terms of a general principle that "the full cause is equivalent to the entire effect" (the Full Cause Principle). Applying it to this case, with the height h through which a pendulum bob can be raised interpreted as an effect that is absolute, and the force of motion of the bob at the bottom of its swing as its cause, Leibniz was able to identify the force of a body's motion as proportional to mv^2, the living force (*vis viva*). If the force of the body's motion were given instead by mv, this would be insufficient to raise it through the height h, since v is proportional to $\sqrt{h}$. Since, ignoring friction, the force produced by its fall through the height h would also be sufficient to cause the bob to have a motive force of mv^2 at the bottom of its swing, Leibniz proposed the universal conservation of force: each isolated body or system of bodies has its own invariant measure of this force, whether this is manifested as living force or as an equivalent dead force—viz. the capacity a body has for producing motion when it is not yet in motion.

Thus the force of a body's motion should not be equated with the force that produces a change in quantity of (directed) motion, Δmv. If action and reaction are understood in terms of the exchange of quantity of motion, then in a collision either body may equally well be regarded as being the cause, since quantity of motion depends only on the relative velocity of the colliding bodies: "it follows from the respective nature of motion that *the action or impact of bodies on one another will be the same provided the speed with which they approach each other is the same*" (*Spec. dyn.*, GM VI 248).[12] The causes of motion in a given scenario should rather be assigned according to a body's capacity for doing work. Force, understood in this way, has the dimensions of mv^2, whether this is manifested in motion (living force, now called kinetic energy), or remains a potential for producing an equivalent motion in a body not yet moving (dead force, roughly, potential energy). This is one sense in which for Leibniz force is "more real" than motion geometrically understood:

> Motion in itself, separated from force, is only a relative thing, and its subject cannot be determined. But force is something real and absolute, and since the calculation of force is different from that of motion, as I demonstrate clearly, it should not be surprising that nature retains the same quantity of force and not the same quantity of motion. (Leibniz to Arnauld [1688], GP II 133)

Some commentators, interpreting this "absoluteness of force" to refer to mv^2, have claimed that it commits Leibniz to a preferred space in which the body would have the velocity v, even if this cannot be empirically determined.[13] On my reading, however, the velocity *is* empirically determined, but with respect to the "fixed existents" supposed. For example, a pendulum bob will have a force of motion of mv^2 with respect to the bodies constituting the pendulum stand, since

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[12] Cf. Corollary 3 of Newton's Law 3: "*The quantity of motion, which is determined by adding the motion made in one direction and subtracting the motion made in the opposite direction, is not changed by the action of bodies on one another*" (Newton 1999, 420).

[13] Dan Garber claims that there is "a correct frame for determining motion, the frame in which the motions observed are the effects of real underlying forces which are their causes. *But this frame could never be identified*." (Garber 1995, 308). Similarly John Roberts argues that by defining space with respect to fixed existents, "Leibniz commits himself to the existence of absolute speed" (Roberts 2003, 561). His argument entails that "mv^2 is a conserved quantity in any inertial frame, but it leaves open the possibility that there is no way of empirically determining the true *vis viva* of any body or of establishing a reference frame that attributes to each body its own absolute speed" (Roberts 2003, 567). Roberts also claims that "mv^2 is an empirical quantity, an arithmetical combination of features of extended matter and its modes, yet Leibniz claims that it is not phenomenal, but metaphysical and real." (562) This is a mistake: derivative forces such as mv^2 *are* phenomenal, for Leibniz, in the (Platonic) sense that they are ephemeral, even though they are modifications of the primitive forces which constitute substances, and therefore real. See Arthur 2014, chapter 6.

these are assumed to be bodies "in which there has been no cause of any change of the order of their coexistence with others" (LC 70). It is true that if the whole apparatus were moving at a speed *v* in the opposite direction with respect to some other system of bodies *S*, the bob would have a motive force of zero relative to *S*. But that would not be relevant to the bob's motion being used as the cause of raising it through the height *h*, and the identification of the cause is with respect to the relative motions of the bob to the stand, not to *S*. The force, moreover, is absolute in the sense that with respect to *S* the bob would have a dead force of an equivalent value to mv^2, even though $v = 0$.

But there is a second sense in which Leibniz held that motion is less real than force. Like time, motion never exists as a whole, since it does not have coexistent parts.[14] "And so there is nothing real in motion but that momentaneous something which should consist in a force striving for change" (*Spec. dyn.*, GM VI 235). This instantaneous striving, the monadic *conatus*, is the foundation of the derivative forces, which are identified by Leibniz as the instantaneous modifications of the enduring primitive forces that constitute what is substantial in bodies.[15] Consequently, all that is real in motion is found in it at an instant, in a body's derivative active forces, whether living or dead. Moreover, Leibniz agreed with Descartes that motion at an instant is only rectilinear, inertial motion, and that there can therefore be no acceleration in an instant. This complemented his understanding of the mechanical philosophy, where all change is effected by discrete impacts causing changes in the quantity of motion in successive moments.[16] Thus the circular motion of an orbiting body will result from a succession of impacts from the particles of the fluid. But "since circular motion results only from the composition of rectilinear motions," Leibniz reasoned, "it follows that if the equivalence of hypotheses holds in any rectilinear motions whatsoever, then it will also hold in curvilinear ones" (*Spec. dyn.*, GM VI 253). So he rejects Newton's centrifugal force as giving a criterion for an absolute motion that breaks the equivalence of hypotheses.

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[14] According to Enzo Vailati, Clarke agreed with Leibniz on this point, arguing that curvilinear motion "is but the Idea of a Number of successive Motions of a Body, never existent together; a pure *Ens rationis*, or operation of the mind (W III, 838)" (Vailati 1997, 126-7).

[15] For a criticism of what I wrote on this in my 1994, see Jauernig 2008; and for a defence, Levanon 2010. Jauernig erects a rococo structure of ontological levels for non-abstract objects, comprising a fundamental level of monads and/or corporeal substances of various kinds, and a phenomenal level with three sub-levels, common-sense, physical and dynamical. For an alternative analysis, see chapter 6 on Leibniz's dynamics in Arthur 2014.

[16] It also complements Leibniz's understanding of how to apply the calculus to physics: see Bertoloni Meli 1995, and Arthur 2013a.

Here, however, Leibniz is guilty of a subtle fallacy of composition. The equivalence of hypotheses applies to motion at an instant, for which he reserves the term *motio* (*Spec. dyn.*, GM VI 237), but it does not follow from this that it applies to motion over time (*motus*), in particular, to curvilinear motion. A comparison of the Copernican with the Ptolemaic or Tychonic Hypotheses involves a comparison of suppositions about sustained causes of curvilinear motions through time; and these will not be equivalent, as Leibniz acknowledges, notwithstanding the fact that the EH holds for instantaneous motions. Thus the phenomena cannot be reduced to what is observable at an instant, as Leibniz insisted.

Now Leibniz's commitment to mechanical philosophy is such that he will take neither gravitational attraction nor the tension in a cord as fundamental. Convinced that all that is physically real about motion is what is represented at each instant, he seeks to give a unified explanation of centrifugal endeavour and the firmness or cohesion of the cord in terms of the pressure of an ambient fluid. Thus in his *Dynamica*, after acknowledging Newton's appeal to centrifugal endeavour as a criterion for absolute rotation, Leibniz grants that "these things would be so if this were the nature of a cord or firmness, and therefore of circular motion"; but, he writes, "in nature there are no cords other than these laws of motion themselves". This leads him into very muddy waters, but we need not follow him there.[17]

2. Newton's "true motion"

Turning now to Newton's views on the relativity of motion, I believe that much clarification can be gained by seeing how they develop out of his criticisms of Cartesian relativism. For despite the profound conceptual innovations Newton accomplished in his *Principia*, I think there are some remarkable continuities in his philosophy of force and motion that are obscured by reinterpreting everything in terms of the "Newtonian mechanics" that was honed by eighteenth century European mathematicians. Chief among these, I shall argue, is his conception of the true motions of bodies as those generated by forces impressed upon them, resulting in a change of their quantity of (directed) motion.

Newton gives his most complete discussion of the relativity of motion as an extended digression in a small treatise with other ambitions— curiously, just as had Leibniz. His *De*

[17] GM VI 507-508; cf. also the similar account in *Spec. dyn*. GM VI 252-53. For discussion, see Stein 1977, Slowik 2006, and, for an admirable attempt at defending his views, Jauernig 2008.

gravitatione et aequipondio fluidorum was intended as a treatise on hydrostatics, but after some initial definitions it turns into a long polemic against Descartes's views on motion that takes up a good two thirds of the manuscript.[18] Scholars have dated the manuscript as early as 1664, and as late as 1684-85; I conjecture it is from 1679, although nothing in my argument here depends on this. *Motion* is defined as "change of place" (def. 4), and *place* as "a part of space that something fills adequately" (def. 1), body being "that which fills place" (def. 2). [19] With these definitions, he writes, he has "determined motion with respect to the parts of this space, not with respect to the position of contiguous bodies" (Hall and Hall 1962, 91-92), as had Descartes. This position conforms very well to that described by Leibniz in 1677 as the one he himself had previously adopted: "the nature of *matter* is to fill space, and *motion* is change of space", where space is an absolute space with respect to whose parts all motions are completely determinate. This consilience is probably a sign of the influence of Gassendi on both thinkers, refracted in Newton's case through Charleton and Barrow.[20]

Now Newton is of course well aware that his definitions in *De gravitatione* are in stark opposition to Descartes's views, and it is in order to forestall any charge that he is gratuitously opposing them that he launches into his critique of Descartes's accounts of space and of motion. I will not enter into the details of his critique, the main point of which is to point out the inconsistencies of Descartes's definitions.[21] The planets are said to be carried by the gyrating vortex around the Sun in generic space, the "common sense of motion", but not in the "philosophical sense", according to which motion is "the translation of one part of matter or one body from the neighbourhood of those bodies immediately contiguous to it, and which are regarded as at rest, to the neighbourhood of others" (*Principles*, Part II, §2 and III §28). Yet the rotating of bodies around the Sun in the philosophical sense "does not cause a tendency to recede from the centre, which a rotation in the common sense can do" (Hall and Hall 1962, 93). The upshot, Newton contends, is

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18 *De gravitatione* was first published in 1962 by Rupert and Marie Boas Hall, both in the original Latin and in English translation.

19 Cf. "The Lawes of Motion", a manuscript from the late 1660s: "There is an uniform extension, space, or expansion continued every way without bounds: in w^{ch} all bodyes are, each in severall parts of it: w^{ch} parts of Space possessed & adequately filled by ym are their places. And their passing out of one place or part of space into another, through all ye intermediate Space is their motion." (Hall and Hall 1962, 157)

20 In his (1970) Edward Strong notes similarities between Barrow's views on space and Newton's, but underestimates their differences, as well as the extent to which these similarities have their source in Gassendi's *Animadversiones*, of which Charleton's *Physiologia* is a translation and amplification – see Arthur 1994.

21 These have been well covered in the literature. See in particular Stein 1977, 2002.

that "the definitions of places, and thus of local motion, [must] be referred to some immobile being such as extension alone, or space insofar as it is regarded as something truly distinct from bodies" (98). For

> unless it be conceded that there can be but a single physical motion of any body, and that the rest of its changes of relation and position with respect to other bodies are just external denominations, it follows that the Earth (for example) endeavours to recede from the centre of the Sun on account of a motion relative to the fixed stars, and endeavours the less to recede on account of a lesser motion relative to Saturn and the aetherial orbit in which it is carried, and still less relative to Jupiter and the swirling aetherial matter which, although not bearing planets, are closer to the annual orbit of the Earth, and indeed, relative to its own orbit it has no endeavour, because it does not move in it. (95)

Since all these endeavours and non-endeavours cannot coincide, Newton argues, "what should rather be said, is that only the absolute motion of the Earth, thanks to which it endeavours to recede from the Sun, coincides with its unique natural motion, and that its translations with respect to external bodies are but external denominations" (94).

Here it is crucial to note that the unique natural motion of a body is caused by a force, either an external principle impressing motion or an internal principle conserving it:

> *Definition 5.* Force is the causal principle of motion and of rest. And it is either an external one that generates or destroys, or at least in some way changes, the motion impressed in some body, or it is an internal principle by which the motion or rest bestowed on a body is conserved, and by which any being endeavours to persevere in its state and resists being impeded. …
>
> *Definition 7. Impetus* is force insofar as it is impressed on a thing.
>
> *Definition 8. Inertia* is the internal force of a body preventing its state from being easily changed when an external force is applied. (114)

According to this account, then, if a body has really been moved, it will have had a motion impressed upon it in absolute space, and the force of this motion will be proportional both to the body (i.e. its magnitude) and to the velocity it gains. Consequently, "the absolute quantity of motion is what is compounded from both the velocity and the magnitude of the moving body"

(115).[22] This is essentially the account of force Newton had developed from Cartesian origins in the *Waste Book* of 1665, which in turn is well summarized in *The Lawes of Motion*:

> But ye motion it selfe and ye force to persevere in yt motion is more or lesse accordingly as ye factus of ye bodys bulk into its velocity is more or lesse. And yt force is equivalent to that motion wch it is able to beget or destroy. (Hall and Hall 1962, 157)

Notably, there is no mention in *De gravitatione* of the main innovations associated with the new conceptualization of gravitation suggested to Newton by Hooke in 1679, and which form the backbone of the various drafts of *De motu* that enabled him to derive the inverse square law: the *vis centripeta* defined in Definition 1 of *De motu corporum in gyrum*, and the proportionality of the space traversed by a body under this force to the square of the time, asserted in its *Hypothesis 4* (Herivel 1965, 257-8). But in the *Principia* this centripetal force is still conceived as an impressed force that generates motion in a given time (Newton 1999, Definition 4, 405), in distinction from the force of inertia. Newton writes: "a body perseveres in any new state solely by the force of inertia" (405), the force of inertia being "the inherent force of matter or power of resisting by which every body, so far as it is able, perseveres in its state either of resting or moving uniformly straight on" (Definition 3, 404). This is changed by the action of an impressed force, which is compounded with the body's inherent force according to the parallelogram law (Law 2, Corollary 1, 417-18). Here the account is supplemented by the all-important novel treatment of centripetal force, the motive quantity of whose measure "is proportional to the motion which it generates in a given time" (Definition 8, 407). Since this is a continuously acting force, however, the time intervals during which it produces this measure are shrunk to zero in a limiting process, yielding an instantaneous acceleration. For Newton, *contra* Leibniz, there *is* acceleration in an instant, and this suffices to break the equivalence of hypotheses: "the truly circular motion of each revolving body is unique, corresponding to a unique endeavour as its proper and sufficient effect" (Newton 1999, 413).

Nevertheless, I contend, this unique circular motion is an example of the unique natural motion Newton had described in *De gravitatione*. In both cases the *true motion* of a body in absolute space is the motion that has been generated in it by the cumulative action of impressed forces. Newton clearly believed that there is a matter of fact about whether a body is actually in motion in absolute space, and what its actual absolute quantity of motion will therefore be. His claim in *De gravitatione*

[22] Cf. *Principia*, Def 2, 1999, 404.

that there is "but a single physical motion of any body, and … the rest of its changes of relation and position with respect to other bodies are just external denominations" (Hall and Hall 1962, 95) is echoed in the *Principia*, where he writes in the Scholium to the Laws of Motion: "True motion is neither generated nor changed except by forces impressed upon the moving body itself, but relative motion can be generated and changed without the impression of forces upon this body" (Newton 1999, 412). Thus the true motion of a given body, generated by all and only those motions impressed on it since Creation, will depend on its cumulative history.

Of course, it is usually emphasized that according to Corollary 5 of the Laws of Motion in Newton's *Principia*, it is not possible to tell whether a given body is at rest in absolute space or moving inertially in that space, since its motions relative to other bodies will be the same in the two cases: "When bodies are enclosed in a given space, their motions in relation to one another are the same whether the space is at rest or whether it is moving uniformly straight forward without circular motion" (423).[23] But this concerns our ability to determine whether there is absolute motion, not whether it exists in fact. As Newton writes in a draft of *De motu* (1684?),

> The motion of a body is its translation from place to place, and is consequently either absolute or relative according to the kind of place. But absolute motion is in fact distinguished from relative in rotating bodies by the endeavour for receding from the centre, which while null in a purely relative rotation, may be very large in one relative to bodies at rest, as in the celestial bodies which are to the Cartesians' mind at rest, although they endeavour to recede from the sun. The fact that this endeavour is always certain and determinate argues for there to be some certain and determinate quantity of real motion in individual bodies, in no way dependent on the relations, which are innumerable and constitute as many relative motions. (Herivel 1962, 305)

One oddity of this account deserves attention. Given his official position that "the causes that distinguish the true motions from the relative ones are the forces impressed upon bodies to generate motion" (Newton 1999, 412), it may seem a little strange that Newton insists on *centrifugal* endeavour as a criterion for absolute motion. For on his own analysis of motion under a central force, it is not a centrifugal but a *centripetal* force that is impressed on a rotating body. Nevertheless, even though there is no impressed centrifugal force, there is still a real and measurable effect of rotational motion. For Newton this is an effective endeavour away from the

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23 Indeed, according to Corollary 6, it is also impossible to tell whether a given body is at rest in absolute space or moving inertially for bodies "urged by equal accelerative forces along parallel lines" (Newton 1999, 423).

centre that is balanced by the endeavour constraining the body in its orbit, whether this is a tension in a string, a pressure from the surrounding wall, or an attractive force from the centre. In modern (neo-Newtonian) terms, there is only one force, the centripetal one, by the continuous action of which the body is continuously deflected from its inertial (force-free) motion. But in Newton's own terms, the centrifugal force is a manifestation of the body's innate force at any instant, by which it endeavours to continue its instantaneous motion along the tangent when a force is applied to deflect it from this path. In the application of the parallelogram law, the centripetal and inertial forces (proportional to the motions they would independently produce in a given time, and to the lines they would respectively trace in that time) are composed to give the resultant motion. The innate force of an orbiting body is thus understood as resulting in an effective centrifugal force that must be balanced by a centripetal force in order to keep the body in orbit around a centre.[24] In Newton's own words in the *Principia*,

> A stone whirled in a sling endeavours to leave the hand that is whirling it, and by its endeavour it stretches the sling, doing so the more strongly the more swiftly it revolves; and as soon as it is released, it flies way. The force opposed to that endeavour, that is, the force by which the sling continually draws the stone back toward the hand and keeps it in an orbit, I call centripetal, since it is directed toward the hand and keeps it in orbit. And the same applies to all bodies that are made to move in orbits. They all endeavour to recede from the centres of their orbits, and unless some force opposed to the endeavour is present, restraining them and keeping them in orbits and hence called by me centripetal, they will go off in straight lines with uniform motion. (Newton 1999, 405)

Another important aspect of Newton's analysis of a body orbiting under gravitational attraction is revealed in the Queries in his *Opticks*. This is his understanding of gravity as an *active principle*. The force of inertia, he explains, is a *passive principle* governing persistence and changes in motion according to the three laws of motion, which cannot by itself produce any new motion. But "by this principle alone there could never have been any motion in the world", and now that there are bodies in motion, "some other principle is necessary for conserving that motion" (LC 176). For there must be something that compensates for the loss of quantity of motion in inelastic collisions,

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24 See Bertoloni Meli 2006 for a careful analysis. Leibniz understands centrifugal endeavour in the same way: "Since every moving body which describes a curved path endeavours to recede along the tangent, one may call this the *endeavour of ejection* [*conatus excussorium*], as in the motion of the sling, for which an equal force is required which restrains the moving body from flying away. *This endeavour may be measured by the perpendicular from the succeeding point on the tangent to the unassignably distant preceding point.*" (*Tentamen de motuum coelestium causis*, GM VI 152; Meli 1993, 132).

as a result of which "motion is much more apt to be lost than got, and is always upon the decay" (176-77). There is therefore "a necessity of conserving and recruiting it by active principles, such as are the cause of gravity, by which planets and comets keep their motions in their orbs, and bodies acquire great motion in falling" (178).[25] In circular motion, for example, the absolute quantity of motion of a body is conserved and its direction of motion is changed at each instant by the action of the gravitational principle, while in free fall it is progressively increased by the same increments at each moment.

There is a necessity also for God to act directly on bodies, both in order to perform miracles and to conserve the regularities of the solar system to compensate for irregularities arising from the interaction of comets and planets, "which will be apt to increase till this system wants a reformation" (180). Such uniformity must be "the effect of choice", and "the wisdom and skill of an ever-living agent, who being in all places, is more able by his will to move the bodies within his boundless uniform sensorium, and thereby to form and reform the parts of the universe, than our spirit, which in us is the image of God, is able to move the parts of our bodies" (181). As Clarke makes abundantly clear on Newton's behalf in his fifth reply to Leibniz, divine action, as well as the actions we perform of our own free will, require changes in the quantity of motion in the universe, which they saw Leibniz as inconsistently denying by his assertion of the conservation of force:[26]

> Indeed, all merely mechanical communications of motion, are not properly action, but mere passiveness, both in the bodies that impel and that are impelled. Action, is the beginning of a motion, where there was none before, from a principle of life or activity: and if God or man, or any living or active power, ever influences anything in the material world; and everything be not mere absolute mechanism; there must be a continual increase and decrease of the whole quantity of motion in the universe. Which this learned gentleman frequently denies. (LC 110)

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25 Newton adds: "It seems to me farther, that these particles [of original matter] have not only a *vis inertiae*, accompanied with such passive laws of motion as naturally result from that force, but also that they are moved by certain active principles, such as is that of gravity" (LC 179).

26 See in particular the long footnote Clarke adds to his Fifth Reply, no doubt in close consultation with Newton (LC 121-25), where Leibniz's force is construed as "the relative impulsive force of bodies in motion", so that its conservation is misrepresented as the conservation of quantity of motion, "which this learned gentleman frequently denies" (110). Action for Newton and Clarke requires the giving of a new force to a thing by an agent (Clarke, 4th Reply, §33/LC 51), and cannot be merely mechanical, as Leibniz maintained. So Newton could not have countenanced a universal conservation of energy such as Leibniz maintained, notwithstanding ingenious Whiggish reconstructions of his words using modern classical mechanics, such as Peter Guthrie Tait's in (Tait 1877, 64-65); see also Newton 1999, 119 for I. B. Cohen's discussion of this.

Thus Newton's denial of the adequacy of a physics based on a merely relative motion is intimately tied in with his views on the nature of divine and human action, and the necessity for these and other active forces to produce increments in the quantity of motion possessed by each body as it charts a unique path through absolute space.

3. Conclusion

We have seen that despite their stark disagreement about centrifugal force as a criterion for absolute motion, there is substantial agreement between Newton and Leibniz on several issues concerning the relativity of motion: that the motions of bodies in a relative space determined by their mutual situations will be the same whether the space is at rest or moving with a constant velocity in a straight line; that the force of inertia governing persistence and changes in motion (in accordance with the laws stated by Newton) is a passive principle, and that because a quantity of motion is lost in every inelastic collision, there need to be *active principles* in the universe to compensate for this loss of motion; that motion understood as change of situation is a mere *external denomination*; that *true motions* are to be distinguished from merely *apparent* ones by reference to the *causes* of the motions, explicated through a consideration of forces; and that this provides a sufficient basis for defending the Copernican hypothesis as true.

But they disagree about how to characterize true motion. For Newton, the true motion of a body is a motion in absolute space, generated by the action of impressed forces, which result in changes in its directed quantity of motion. Losses of absolute quantity of motion in inelastic collisions will be compensated by gains introduced by active principles such as gravity, fermentation and cohesion, and by the actions of mind on matter, as in the case of our or God's willed actions. Although the absolute quantity of motion in a body resulting from these actions is not empirically determinable because of the indiscernibility of rest from inertial motion, true circular motion (and indeed any curvilinear motion around a centre of force) is distinguishable from merely apparent motion by the presence of a centrifugal endeavour away from the centre.

For Leibniz, true motions are identified by reference to the most intelligible hypothesis. He rejects the conception of true motion as change of place in absolute space, believing that situation and space must be determined relative to certain bodies considered as immobile. Moreover, because the equivalence of hypotheses applies in every collision, he believes that causes cannot therefore be identified with impulsive forces, but instead require determining through "the providing of a reason" (A VI iv 1620). True motions are assigned according to the most intelligible hypothesis as to which bodies should be taken as immobile in order to account for a given system of phenomena. This connects with Leibniz's construal of space as relative to certain bodies considered as immobile, on which basis he aimed to construct a mathematical space adequate for representing and predicting those phenomenal motions. Such a space, however, is not a really existing thing, but a construction for the purposes of representing the motions in a given system of phenomena. Meanwhile the apparent loss of motion in inelastic collisions is for Leibniz only an appearance, and active force (of dimension [mv^2]) is conserved in every isolated body, whether this force is manifested in motion (living force), or potential (dead force). So it is not living force itself that Leibniz conceives as invariant, but the total active force in each isolated system, whether manifested in motion or stored up in dead forces such as centrifugal force and elasticity. Newton rejects such a universal conservation of force as precluding any action of an immaterial substance on matter, rendering all motions determinable in principle by the laws of mechanics alone. But this is precisely Leibniz's reason for advocating it.

Leibniz is mistaken, however, in rejecting centrifugal endeavour as a criterion for absolute circular motion. As we have seen, his argument that the equivalence of hypotheses (EH) applies to curvilinear motions involves a fallacy of composition. From the fact that EH applies to motion at an instant (Leibniz's *motio*), it does not follow that it applies to motion over time (*motus*), in particular, to curvilinear motion. EH in the former sense is the relativity of (instantaneous, local) motion that holds in special and general relativistic physics, just as much as in classical physics. But, in modern terms, you cannot transform away a rotation by adopting a different reference frame in any of these theoretical frameworks without leaving unresolved forces or appealing to adventitious causes—thereby producing, even on Leibniz's own terms, a less intelligible hypothesis.[27]

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[27] Notwithstanding the equivalence in GR between gravitational acceleration and free fall towards a central mass in a correspondingly curved region of spacetime, such a (spherically symmetric) spacetime metric becomes distorted if the mass is rotating, and a radically different effect would be produced by the rotation of all the other bodies around it instead. It is not correct, therefore, to equate General Relativity with a universal relativity of motion, as does Jauernig, for example: "no experiment can distinguish between different frames of reference, regardless of their state of motion (General Relativity)" (2008, 2).

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