Mechanical Principles

2. Translation[1]

We have resolved to set down the principles of the science of motion called Mechanics. The principles should be clear, certain, few and sufficient for the explanation of the rest. When these have been constituted in each science, the rest can be accomplished almost by a kind of calculus, and reduced to a superior and simpler science. For just as problems in geometry /102/ are reduced to arithmetic by having recourse to just a few propositions from Euclid, so what we need to bring about is that when our principles are understood the difficulties of mechanics should be reduced to pure geometry. For in this way it will be no more difficult to judge the success of machines than the truth of certain geometrical theorems. And when the art of enumerations called combinatorics is rendered easier by the intervention of a rational characteristic, we may believe that even the invention of machines will be no more difficult than the construction of geometrical problems.

Nor should we hope that natural science can be greatly advanced before [this is done]. For just as mechanical principles [*rationes*] are derived from geometry, so physical principles are derived from mechanics, and as long as it is difficult to invent machines suitable for human use, or to explain discoveries people have made, we will try in vain to make known the causes of natural things and specimens of the divine art in the phenomena of nature.

Since, however, even someone knowing principles may not know how to apply them to examples unless he sees it done by others, we will therefore, whenever we treat mechanical principles, give various specimens of the science illustrated with examples chosen in every kind of motion. The writers on the Elements of Mechanics are accustomed to treat scarcely more than the theory of equilibrium and the five powers, by which a smaller power can balance one twice as large, but do not mention impacts and collisions, the firmness of solids, the resistance of media, the tensions of a bow, the flow of fluids and many other things of that kind. But because the contemplation of the latter things, while a little more abstruse, is no less pleasing and useful, we will therefore try to remedy their deficiency, at least enough to open up the way for others.

In order to understand thoroughly the Principles of Mechanics or laws of motion, it will be useful to lead the reader through an outline of our inquiries. Whenever we observe something to move, /103/ we notice first of all some change or state of things different from what we remember sensing shortly before. In fact this difference consists in the situation of the bodies, for

if there occurs only a change in the quality or forces of the bodies, such as a change in colour, heat, or weight, we do not sense motion in it, even if there is perhaps some in it. Situation is a certain state of bodies by the recognition of which among certain bodies that are given or found, we can detect and attain to some other body; or, situation is a mode according to which any body can be found, even though we recognize nothing in it specifically by which it can be distinguished from the others. This way of finding the body depends on knowing its distance from other bodies, and also on knowing the angle, that is, the figure which it makes with another body. For lines determine problems either by themselves or together with angles, as is known from trigonometry. But since figure is recognized by angles, and the angle by the distance from certain points, it is thus clear finally that situation reduces to the recognition of distance, and so change of situation cannot take place without a change of some distance.

Distance is the shortest path from one thing to another; therefore there is no distance if the extrema of two things are together. Hence if we notice things together at one time, and apart at another, we infer that a change of situation has taken place. As for greater or smaller distance, we either infer it from other things already known using a kind of geometrical reasoning whose beginnings are also discerned in uncultured people, or we define it by the magnitude or multiplicity of bodies lying between; that is, if we know from experience that some one constant body can be placed end to end several times between two bodies arranged in a straight line, then this will be the distance measured; and the constant body is called the measure. In order not to keep needing a new determination, we generally make this measure out of some durable matter, since there cannot be a memory of its quantity per se unless it is made observable by certain qualities. If there is no other measure, people usually use their own body /104/ and its parts for a measure. Whence people have used the pace,[2] the foot, the palm of the hand and the finger for measuring things, and just as when we handle things we judge them to be bigger the bigger the part of the hand they occupy, so for those things whose image reaches the back of the eye, we estimate their size by the size of the portion of it they fill. From the size of the image and the magnitude of the distance of some body whose size is already known, or of that between two points seen at the same time, we estimate distance. But if we now notice a change in these things, we declare that some motion has taken place.

However, change of situation is not yet sufficient for us to judge which of two things that have changed situation with each other we should ascribe the motion to. Thus we must enquire

separately into the reasons for this judgement. At any rate people seem to make such judgements, since they sometimes ascribe motion to one or the other of two bodies, and sometimes to both, and consequently distinguish motion from change of situation. For example, when they sail from port they ascribe motion to the ship rather than the earth, because they see the sails billowing with the wind, and the sea foaming under the impacts of the oars; and when they go for a walk, they believe they are approaching the town, rather than the town approaching them, because they feel some fatigue and exertion in themselves, and besides are accustomed to attributing motion to small things rather than big ones. From these things it is clear that in the case of two bodies, motion is attributed to that one which contains the cause of their mutual situation having changed, because we have seen it receive a blow, or because it is dislocated and deformed, or shows other signs of having received blows and of the change made in it as a result. If such signs are absent, we judge from what could happen more easily, or from what has usually happened up till now. So we are in no doubt as to whether it is the stagecoach that moves and not the tree, for the tree is connected to the field by its roots, the field makes up part of the earth's surface; but no one doubts /105/ that the coach moves over the ground rather than the ground under the coach. If there are no reasons to the contrary, we ascribe motion to other things rather than ourselves. Thus it is believed that the stars turn around the earth every day, rather than that the earth turns on its own axis; for at any rate the motion of the stars is apparent, and we ought not to depart from the apparent without a reason. But to ordinary people the reasons of the philosophers have not been able to come to mind.

The philosophers, on the other hand, have had weighty reasons for the contrary opinion. For having understood the immense distance of the stars, they believed it was more in keeping with reason to attribute a modest motion to the earth, like that found in most everyday bodies in our era, than that the observable universe should be moved with some insane rotation. Moreover when they discovered that the heavens were fluid they easily recognized that the celestial bodies were not fastened together by any firm bond, so that they could not be easily moved with one common motion. Also the beauty and simplicity of the Copernican System easily attracted all the most talented people onto its side. For the sun is the largest of the planets, and it seemed right that the source of light and heat, and perhaps motion too, should be located at the center; and with the earth transferred into the place previously ascribed to the sun, many imaginary circles, many eccentric circles, and many anomalies vanished. But if also the apparent diameters of the

fixed stars change, or different parts of the heavens are vertically above the same point of the earth at different times, and this agrees with the hypothesis of the annual motion of the earth, according to which the earth changes its situation to the fixed stars, this hypothesis would certainly seem to be sufficiently corroborated. In addition there is another new[3] indication from the motions of the earth, for if it is certain that during earth-motion hanging lamps constantly oscillate from east to west, or that waves impinge only on eastern and western shores, /106/ this is a credible indication that there is some diurnal motion of the earth, for a general motion can be rendered sensible when some part of the earth moves in an extraordinary way.

Sometimes it happens that in order to explain a change of situation it is necessary to attribute the motion to two bodies, by the composition of whose motions the apparent motion can be saved. And [this composition] is very often used to distinguish two motions from each other, and to define what speed and direction each one contributes, so that if a ship is carried by the wind and the currents at the same time, and if we can always tell which part of the motion each one contributes, we will be able to estimate the course of the ship and find where it will arrive. For the force of both the wind and the currents will be estimated separately. The stronger the wind is, the more it inflates the sails, and the faster the water flows, the higher it washes up above the shore, all of which must be determined by subtle estimation.

In other respects, someone could regard the question as quite empty, and as signifying nothing about diurnal motion or the earth's being at rest, and nothing at all about the absolute speed of any body. For it could be judged that motion is something respective, so that when things are changing situation among themselves, it makes no difference which of them it is attributed to. Thus suppose there are only two bodies *A* and *B* in the world, approaching each other and passing from the situation *AB* to the situation *(A)(B)* with a uniform velocity, and in some determinate time; let this be as *AB*, the magnitude of which expresses the *speed* of approach, ôr change of situation, which some call *respective*. Then it is impossible for anyone to determine which of the bodies taken individually is in motion, and just which of their /107/ absolute speeds and directions is to be expressed by the lines *A(A)* and *B(B)*.

[copy editor: insert figure 1 here]

For we can imagine, as in *figure 1*, that, while body *B* remains at rest, body *A* alone is carried towards it and that all the speed is to be ascribed to it, so that the straight line *A(A)* (the distance between the two places *A* and *(A)* of the body *A* which represents the absolute speed and

direction of the body *A*) is equal to *AB* (the distance between the two mobiles, which expresses the speed of its approach), and likewise the direction (that is, *(A)* and *B* are in the same direction from *A*). But by the same right that this is said about *A* or *(A)*, it could be said about *B* and *(B)* of *figure 2*, and vice versa.[4]

[copy editor: insert figure 2 here]

The former explanation of the phenomenon will seem true to someone stationed in body *A*, the latter explanation to someone else who will be in *B*, for assuming motion to be above all equable and not impinged on by anything external, he who is in *A* will attribute perfect rest to himself, and motion to the other, and he who is in *B* will do the same thing. But it can turn out that both are wrong, and that the same phenomenon will be explained by assuming another hypothesis. In fact, infinitely many hypotheses can be assumed.

[copy editor: insert figure 3 here]

For in *fig. 3* if we divide the speed equally or even in some proportion between the two bodies, provided the sum of the speeds in each case is equal to that which we attributed to one whole in the first or second hypothesis, the same appearances will always be produced as before for those stationed in *A* or *B*. For if the speeds of the individuals are as the sections *A(A)* and *B*(*B*) of the straight line *AB* (ignoring the interval between *(A)* and (*B)*, that is, imagining the points *(A)* and (*B*) to coincide or assuming the bodies *A, B* to be minima), then the two bodies will collide together at *(A)*(*B*). But the person stationed in *A* will notice only the approach of *B* towards him, and will therefore ascribe the whole motion to it.

[copy editor: insert figure 4 here]

The same will be the case *in the fourth figure*, even if the two bodies *A* and *B* do not tend towards one another, but one flies away and the other follows, provided that (as before the sum, so here) the difference *AB* of the speeds *A(A)*, *B(B)* is the same as that which we attributed to one of the two bodies alone *in the first and second figure*. /108/ For in this way, subtracting from each the equal common velocity by which they tend in the same direction, the body *A* that is following will approach the one flying off with the excess of speed; and this approach will be all that appears to the person stationed in *B*. In the same way, provided no reason can be discerned in circular motions, supposing two bodies alone, the earth and the heaven, whether the heaven is carried with a diurnal motion from east to west, or instead the earth with a contrary motion from west to east. But if therefore there is clearly no way one can be discerned from the other, and if

not even he who has examined and carefully explored everything could notice any distinction, it will follow that they are both the same, and that the question concerning absolute motion and rest is empty, and signifies nothing.

[copy editor: insert figure 5 here]

Of course, if an eye were put in some third body that was established as lacking motion, it would indeed be clear from this to which of *A* and *B* motion should be attributed. But how could it be established that the third body lacked any motion at all, when it seems that no indication of absolute rest can be had? Let us see, however, in how many ways the same phenomenon can be explained by assuming a third body, and whether with the aid of this assumption something certain can be determined concerning the absolute and proper speed of bodies. Let us suppose, then, that to an eye assumed at *C* two bodies *A* and *B* appear to approach each other, and that everything seems equal on both sides, namely at the beginning and at any intermediate time, and that at the end of the motion the situation of *A* and likewise of *B* will always appear the same with respect to *C* and the other bodies around it that are firmly connected to *C*. Then *C* will of course appear to be at rest with those it is connected to, whereas the bodies *A* and *B* will appear to move with equal speed and tend towards one another along the straight line *AB*, until they collide at *(A)*(*B*). But in order for us to be certain of the absolute truth of this appearance, we must inquire whether we cannot satisfy the same phenomenon by making another supposition, /109/ namely by attributing some motion to the eye *C* itself.

[copy editor: insert figure 6 here]

Let us then ask whether it is possible for the appearances I spoke of to be saved if body *A* is assumed to be at rest, and body *B* and the eye *C* are both moved, so that anyone must attribute motion to them in order for *A*'s resting to be reconciled with the appearance. And how this can happen I will show as follows. Suppose *B* is carried with a speed and direction *BA* and the eye *C* with a speed and direction *C(C)*, parallel to *BA* and similarly directed, but equal to *B(B)* and *A(A)*, that is, to half *BA*. Assuming these things, it will appear the same as when the eye *C* was at rest, and *A* and *B* appeared to be moving equally. This is demonstrated because, with this supposition made, *A* will seem to approach *B*, *B* to approach *C*, and *C* to approach *A,* in the same way as before: *A* will seem to approach *B* as before, since *A* and *B* come from a distance *AB* to nearness or contact at *(A)*(*B*) by a direct equable change, with a speed of approach proportional to *AB*, for it does not matter to explaining the approach of *A* and *B* whether we ascribe the whole

velocity *AB* to one of them, or to both parts—*by the preceding paragraph, cases 1 and 2*. Similarly, *B* seems to *C* to approach in the same way as before, for *C* flies off towards *A(C)* with a speed proportional to *C(C)*, half of *BA*, but *B* will follow with the total speed *BA*. Therefore *by the preceding paragraph, case 4*, *B* will approach *(B)C* with the difference of speeds, that is, with speed and direction *B(B)*, which is half *AB*, in short, as in the supposition of *C*'s being at rest. Finally, *A* also seems to *C* to approach in the same way as before, as is obvious, because it makes no difference to the approach, *by the preceding paragraph, cases 1 and 2*, whether with *C* at rest, *A* arrives at *A(C)*, or with *A* at rest, *C* arrives at *A(C)*.

It is clear then that even if a third place is assumed for the eye *C*, nothing determinate is thereby established about the proper and absolute speed of each of the three bodies *A, B, C*, but everything remains uncertain and indeterminate. For whatever speed or direction we attribute by assuming an absolute motion for one of the bodies, we will always find that anyone must then understand motion in the others in such a way that everything will appear as before. But even if many more bodies still are assumed, and the eyes in each of them are collocated, this will be discerned to happen. And this contemplation, which it will be enough for us to have begun in this place, is neither inelegant nor useless. /110/

Hence it can truly be understood that whatever multiplicity and variety of bodies are supposed, and however many observations are made, it is still never possible to demonstrate which motion is the absolute and proper motion in bodies—indeed, not even the least determination can be found for excluding any of the various possible hypotheses. Hence it is no wonder that talented men have been able to devise a huge number of Astronomical Hypotheses, and have supposed every possible Planet, even the Moon if it pleased the gods, to be located at rest at the center of the world. And since a multiplicity of bodies and observations does not decide, it follows that the same thing will happen even if the number of moving bodies is infinite—or at any rate, more than can be enumerated by human diligence. Just as happens when a solid moves in a liquid, whether in a straight line or around its own center. For when it is moved in a straight line it is necessary that the liquid is divided into innumerable parts, each of which is endowed with its own particular motion, in such a way that returning in a circle it fills up the place deserted by the solid. But when a solid body is rotated around its own center it rejects contiguous bodies along the straight line that is tangent to the circle of the motion. From this, however, it does not follow that it can be determined with absolute or mathematical certitude that the solid body is moving

rather than at rest, since one may always imagine various compositions of motion in the parts of a liquid through which the same phenomena will be explained with the solid at rest; even if these suppositions are remarkably complicated [*perplexae*], and that is the simplest which rather attributes motion to one solid and derives from it the motion in the parts of the liquid.

From this it is therefore clear that from the phenomena of the changed situation alone no certain knowledge can ever be had concerning absolute motion and rest. But if absolute motion cannot be distinguished from other phenomena, not even by him to whom all the phenomena are revealed, it follows that motion and rest taken absolutely are empty names, and whatever is real in them consists in respective change alone. For since no hypothesis can be refuted rather than others through certain demonstration, not even by someone omniscient, it follows that none is false rather than others; that is, (since they cannot be consistent) they are all false; nor can they be accepted except as various /111/ appearances of the same thing, or optical tricks, according as the eye is stationed in one place or another. In this case, however, we will be permitted to choose the simpler mode of explaining, which involves reference to a cause from which the remaining changes may be derived more easily. Hence we will not hesitate to attribute motion to the solid body from which we can deduce the undulations of the surrounding liquid, rather than thinking of those undulations as originative; thus we will say that the stone falls to the earth rather than that the globe of the earth together with the whole universe leaps up towards it. Even if perhaps the falling of the stone and the leaping up of the earth are equally alien to nature, nothing really happens but a certain respective change of situation or translation from a separate place to a near one. But whether we may assert this will finally appear more clearly only when we know whether the same phenomena will follow from the collisions of two bodies, in whatever way their absolute motion is explicated in the four cases of the *first paragraph*. This will be discussed below.

Indeed, even if this were so, the question whether motion should be attributed to the heavens or to the earth would not therefore be senseless, but would have to be explained in a different way than is usually done. For when the question is perfectly resolved, it will be understood that nothing other than the following sense is possible, whether the fluid matter surrounding the earth has various motions of its parts circling back on themselves, or receding from the earth along the tangent: namely that these things can be explained more distinctly by the supposed motion of the earth and its being reduced to a simple cause. Apart from this, if no such matter is understood to

be surrounding the earth, or if, given such motions of the surrounding bodies, someone wanted to demonstrate that they could not in any way be explained except by supposing the absolute motion of the earth, he would, given what has been said, not be doing anything at all.

N.B. When two bodies rebound from each other after a collision, it is necessary that after the collision there is motion in one or the other of them, or perhaps in both; now it must be seen whether treating the cases differently after the collision will produce a criterion. Supposing nothing certain can be defined without a collision, it follows that after the collision too nothing certain can be defined, for otherwise different entire causes could produce the same full effect. Shape, of course, cannot[5] be produced in many ways, but shape is not something complete.

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[1] A VI, 3, 101-111 (LH IV 3, 5a, leaves 1-4, two sheets of quarto, 8 pages).

[2] 'Pace' translates *passus*, a thousand such paces making a Roman mile.

[3] In the margin Leibniz has written the alchemical sign for distillation, ♃, a sign he used to indicate that the issue in question needed further analysis.

[4] I have made a correction to what seems to be an error in Figure 2, where (in the Academy edition) B is represented on the right by a circle instead of a square, as it should be, and as (B) is.

[5] The "non" here seems to be an error; Leibniz has already articulated the view in 1676 that it is precisely because the same shape *can* be produced in many different ways that we can infer that a merely geometrical body is not a complete thing. See the *Meditatio de principio individui* of 1676: "whoever understands some effect perfectly, will also arrive at a knowledge of its cause. For it is necessary that there be some connection between the entire cause and the effect." (A VI 3, 490; DSR 51).