Mechanical Principles

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1. Commentary

The following is a translation of a short essay by Leibniz, *Principia mechanica*, that has long been available in its original Latin in the *Akademie* edition, in series 6, volume 3, N. 6, pp. 101-111. It appears to have been composed some time between late summer 1676 and February 1677, which is why the Academy editors decided to include it with the *De Summa Rerum* texts. The essay is notable for its extended discussion of the relativity of motion, including several theses usually thought to have originated later, such as the characterization of Copernicanism as the simplest (i.e. most intelligible) hypothesis. But its significance for understanding the evolution of Leibniz's views on relativity of motion has been largely underappreciated by commentators, a state of affairs I hope this translation may do something to reverse.

The piece is introduced by Leibniz as part of his project for a universal characteristic. Once clear principles for mechanics are established, he asserts, it will be no more difficult to understand how machines work—and even to invent machines—than it is to prove geometrical theorems, simply by applying combinatoric reasoning to the definitions and principles. Motion, geometrically understood, is simply change of situation. Situation, on the other hand, is determined by angles and distances. Here Leibniz follows Hobbes, but we can also see him taking some of the first steps toward his mathematical theory of situation, the Analysis Situs, including a definition of situation as "a mode according to which any body can be found, even though we recognize nothing in it specifically by which it can be distinguished from the others" (103), and his characteristic definition of distance as the shortest path between two things.

Almost immediately, however, the essay devolves into an extended discussion of the relativity of motion,[1] in the course of which Leibniz gives one of his clearest explanations of the position he will continue to hold throughout his mature writings, that motion, understood geometrically, is completely relative. He expounds this by reference to six cases. The first four involve two only bodies, A and B. Leibniz shows the same appearances will follow with B at rest and A moving towards it with a uniform velocity v, with A at rest and B moving towards it with a uniform velocity $-v$, with A and B moving along a line towards one another with velocities $\frac{1}{2}v$ and $-\frac{1}{2}v$, and with A and B moving uniformly in the same direction with a difference in velocities of v. In case 5 he considers whether it makes any difference to introduce an eye in a third body C

(assumed to be at rest) observing the motions of A and B, and shows that the phenomena—all the mutual situations at each instant—will appear the same even when C is allowed to move instead along in the same direction as B but with half its velocity, as in case 6. Therefore, Leibniz, concludes, not even an omniscient being will be able to determine which body is in absolute motion: "whatever speed or direction we attribute by assuming an absolute motion for one of the bodies, we will always find that anyone must then understand motion in the others in such a way that everything will appear as before" (109).

All this, however, concerns motion insofar as it is *understood geometrically*, that is, as simply change of situation. Here Leibniz adduces an important distinction, I believe for the first time, that between *motion understood geometrically* and *motion with respect to cause*.[2] For although changes of situation are entirely respective, this is not the case if motion is understood with respect to cause, when it can be attributed to that thing that is the cause: "in the case of two bodies, motion is attributed to that one which contains the cause of their mutual situation having changed" (104). "No one doubts," he notes, "that the stagecoach moves over the ground rather than the ground under the coach", because we know the cause of the motion of the coach, and that the tree with respect to which it moves is attached to the ground, the ground to the planet Earth, and so on. In more complicated scenarios, however, the cause must be identified by reference to the *simplest hypothesis* for explaining the phenomena in question.

This distinction establishes an interesting connection between Leibniz's account of the relativity of motion and his philosophy of causation. Although "not even the least determination can be found for excluding any of the various possible hypotheses" about which particular bodies are in absolute motion (110), still "we will be permitted to choose the simpler mode of explaining, which involves reference to a cause from which the remaining changes may be derived more easily" (111). As he explains in the *Specimen inventorum* of the late 1680s, "that thing from whose state a reason for the changes is most readily provided is adjudged to be the cause" (A VI iv 1620, Ar 311). He even gives the same example here as he will there of a solid moving through a liquid, and the hypothesis of the solid's moving being "infinitely simpler than the others". As he concludes there, "Causes are not derived from a real influence, but from the providing of a reason" (1620).

Of particular interest is the fact that Leibniz uses this distinction between motion as change of situation and motion with respect to cause to defend the Copernican hypothesis. There are many

phenomena by which the hypothesis of the annual motion of the earth "would certainly seem to be sufficiently corroborated" (105). Leibniz cites as evidence not only the far greater simplicity of the Copernican hypothesis in dispensing with the imaginary epicycles and eccentric circles, but also potential changes in the apparent diameters of the fixed stars and changes of situation of the earth relative to the fixed stars; and as evidence for the diurnal motion, "that during earth-motion hanging lamps constantly oscillate from east to west, or that waves impinge only on eastern and western shores" (105). As he concludes, "these things can be explained more distinctly by the supposed motion of the earth and its being reduced to a simple cause" (111).

Obviously, this anticipates the argument Leibniz will give in the tract he writes in Florence in 1689, suggesting a way in which the Italian philosophers "could freely follow Copernicus without damaging the authority of the censors, if only they were to recognize, with us, that the truth of a hypothesis should be taken to be nothing but its greater intelligibility" (C 592, AG 92). He does not go quite that far here, however: he talks of "being permitted to choose" the simplest hypothesis, or of Copernicanism being "sufficiently corroborated", without calling it true. In fact, as Dan Garber has observed,[3] here he claims that "since no hypothesis can be refuted rather than others through certain demonstration, not even by someone omniscient, it follows that none is false rather than others," and that all are therefore strictly speaking false (110). For example, the hypothesis that the Sun is stationary cannot be known to be true, because, by the arguments given here, the phenomena would appear exactly the same at any given instant if the Sun were moving with a constant velocity in a given direction, and if this same velocity were compounded with the motions of all the other "eyes" by which it could be observed. We recognize this as the relativity of inertial motions. Here, years before Newton has even thought of writing the *Principia*, Leibniz is already rejecting the idea of absolute motion on the grounds that it violates the relativity of motion.

Again, prior to Newton's mooting of the centrifugal force arising from circular motion as yielding a criterion for absolute motion, we see Leibniz considering the idea that "a body rotating around its own centre rejects contiguous bodies along the straight line that is tangent to the circle of the motion" (111). But this cannot, contrary to what Newton will claim, establish absolute motion, because at any instant "one may always imagine various compositions of motion in the parts of a liquid through which the same phenomena will be explained with the solid at rest" (111). Nevertheless, "these suppositions are remarkably complicated, and that is the simplest

which rather attributes motion to one solid and derives from it the motion in the parts of the liquid." Thus Leibniz rejects any criterion for establishing absolute motion and rest; but allows that for a given set of phenomena one should adopt the simplest hypothesis as assigning the correct cause. The continuity between the position described here and the official position in the *Dynamica* of 1695 is plain to see:

> Universally, when motion occurs, we find nothing in bodies by which it could be determined except change of situation, which always consists in relation (*in respectu*). Thus motion by its nature is respective. But this is to understand these things in mathematical rigour. Meanwhile, we attribute motion to bodies according to those hypotheses by which they are most aptly explained, and the truth of the hypothesis is nothing other than its aptness. (*Dynamica*, Proposition 19, GM VI 507-508)

What Leibniz fails to appreciate here, though, as he will also later, is that the equivalence of hypotheses may well hold at each instant—that is, for motion taken instantaneously—without that entailing that it will also hold for non-uniform motion over time, such as motion in a circle. In the second (unpublished) part of the *Specimen dynamicum*, he argues against Newton that "since circulation too arises only from the composition of rectilinear motions, it follows that if the equivalence of hypotheses is preserved in any supposed rectilinear motions whatever, it will also be preserved in curvilinear motions." (GM VI 253; AG 136-37). Unfortunately this is a fallacy of composition, although a subtle one. For while Leibniz is correct that circular motion will not yield a criterion for absolute motion and rest, it will still yield a dynamical criterion for the non-equivalence of a body's spinning and everything spinning around it. Granted, Leibniz allows that the former is the infinitely simplest hypothesis. But this is a hypothesis about motion over time, not a hypothesis for assigning absolute motion at any instant, for picking out one rectilinear motion as privileged. There is no relativity of circular motion, and he is wrong to suggest that the equivalence of hypotheses applies to it.

The original is on two sheets of quarto of 8 pages: LH IV 3, 5a. Leaves 1-4. As regards its date of composition, the editors note that it "bears a rather singular watermark. It is not out of the question that Leibniz first drafted it in Hanover. At any rate it was begun later than the foregoing pieces (N.1-N.5), but should be placed in context next to the Paris writings for purposes of discussion." This seems likely: the note at the end appeals to the principle he had formulated in the summer of 1676 that different entire causes cannot produce the same full effect; and a piece

from February 1677 ("Motion is Something Relative", A VI iv, 1970; Ar 229) proceeds as though the distinction between motion understood formally and motion with respect to cause has previously been established. It is also clearly prior to Leibniz's working out of the connection he sees between attribution of cause and the new doctrine of force in 1678-9, discussed by Michel Fichant, who gives other related but later unpublished texts in his edition of the *De corporum concorsu* (*La réforme de la dynamique*, Paris: J. Vrin, 1994). I surmise a probable date of composition of some time between August 1676 and February 1677.

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[1] Curiously, Newton does much the same thing in his essay *De gravitatione* ("On the gravitation and equilibrium of fluids"), which, after beginning as an essay on the principles of fluid mechanics, almost immediately goes over into a long diatribe against the Cartesian account of motion as something merely relative.

[2] This distinction is reiterated in many texts, as early as February 1677 (A VI 4, 1970; RA 229), and as late as the *Dynamica* of 1695, quoted below.

[3] Daniel Garber, *Leibniz: Body, Substance, Monad* (Oxford: Oxford University Press, 2005), p. 109. Garber's discussion should also be consulted for the various nuances and changes in Leibniz's treatment of the relativity of motion over the ensuing years.