The Labyrinth of the Continuum

1. introduction

Leibniz typically described his theory of substance as a solution to the difficulties of the continuum, a way out of the infamous "labyrinth of the continuum". This will seem odd if the composition of the continuum is understood as a purely mathematical problem—that of whether a line, for instance, is composed of points or infinitesimals, or perhaps neither. Natural philosophers in the seventeenth century, however, understood the problem in a much wider sense, as applying to all existing quantities and their composition: what (if any) are the first elements of things and their motions? Are there atoms or indivisible elements of substance, as Gassendi and many of his contemporaries proposed? If so, this leaves many difficulties unresolved: what is the origin of their cohesion? why does the division of matter stop at some point? Perhaps, as Galileo proposed, matter is made up of points, held together by the indivisible voids separating them, with both points and voids lacking quantity. But then how do things lacking quantity compose into something continuous? In Descartes' natural philosophy individual bodies are distinguished by their differing motions, which actually divide them. But in order for there to be motion in the plenum, this division must proceed to infinity. How is this possible? For Hobbes, a body in motion must have at each instant an endeavour or "beginning of motion" (corresponding to a Galilean "degree of motion" or one of Cavalieri's indivisibles), and these same endeavours are the cornerstones of his materialist psychology. But can any sense be made of the composition of motions from endeavours without supposing the composition of space out of points, or time out of instants, which geometry forbids? It is this whole cluster of problems concerning infinite divisibility, the actual infinite, mathematical indivisibles, the existence of atoms of matter or substance, and the analysis of continuous space, time and motion, that constitutes Leibniz's "labyrinth of the continuum", and for which his theory of substance was intended as a solution.

In outlining his solution, Leibniz characteristically explained it by contrast with the false conceptions that he believed had led others astray (and which correspond, in almost every case, to blind alleys from which he had had to extricate himself in his earlier studies). Continuous things, he stressed, cannot be composed by an aggregation of first elements: insofar as anything is continuous, its parts are indiscernible from one another, and thus indefinite. The continuum is therefore not an actually existing thing, a whole composed of determinate parts, but an abstract entity. Thus "those who compose the line from points have quite improperly sought first elements in ideal things or relations".[1] In existing things, by contrast, the parts are determinate, and units are prior to any whole aggregated from them. Matter, for example, considered abstractly, is homogeneous and continuous, consisting in a pure potentiality for division; but taken concretely it is at any instant not only infinitely divisible, but actually infinitely divided by the differing motions of its parts. As a result of these continually changing motions, no part of matter, however small, remains the same for longer than a moment; even shape or figure is evanescent, and a body with an enduring figure is something imaginary. Similarly, there is no stretch of time, however small, in which some change does not occur.

Actual things, on the other hand, presuppose true unities. Thus each actual part of matter presupposes something that is truly one, and from which its actuality derives. These true unities are what Leibniz calls simple substances, entities which contain an "entelechy" or actualizing principle. Leibniz's employment of these scholastic terms reflects his deeply held belief that many Aristotelian principles, when properly interpreted, provide a sound foundation for the mechanical philosophy that is otherwise lacking. In Descartes' philosophy material substance is essentially passive, and so does not contain any foundation from which its motions or capacity to do work can be derived. For Leibniz, a substance is rather essentially something that acts, and the effects of its actions will be local motions. The capacity for such action, married with a tendency to actually bring it about, is what he dubs "force": "and this force does not consist in a mere faculty, such as the Schools seem to have been content with, but instead is endowed with an endeavour or striving, such that it will attain its full effect unless it is impeded by some contrary endeavour."[2] This force is thus the foundation for the motion of any actual part of matter at any instant: "For not only is a body in a place commensurate with itself at the present moment of its motion, but it also has an endeavour or striving to change place, so that the subsequent state follows of itself from the present one, by a force of nature".[3] One sense in which this primitive force is an entelechy, something that "makes something actual", is that the differing endeavours or tendencies to motion to which it gives rise are what make actual the various parts into which matter is divided at different instants. Without some such entelechies or principles of activity, Leibniz held, the various parts into which mechanical

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1 "Remarques sur les Objections de M. Foucher" [1695]; GP IV 491. All translations given here are my own.
2 *Specimen Dynamicum* [1695], GM VI 235.
3 *De ipsa Natura* [1698], §13; GP IV 513.

philosophers supposed matter to be divided at any instant would not even be distinguishable from one another.[4] This necessity for there to be such active substances everywhere in matter is, in Leibniz's eyes, what is missed by his contemporaries such as Arnauld, De Volder or Foucher: "those who have found that relations such as number and space (which comprise the order or relation of possible coexistent things) cannot be formed from an assemblage of points, have for the most part been mistaken in denying that substantial realities have first elements, as if there were no primitive unities in them, or as if there were no simple substances."[5] Simple substances, for Leibniz, are unities that cannot be resolved into anything more primitive: they are enduring sources of activity, from each of which emanates a series of states governed by its own individual law, each state being a representation of everything else in the universe contemporary with it. Each substance, moreover, being a primitive force or source of activity, also involves an instantaneous endeavour or striving towards a change of state, and this endeavour is what is real in motion at each instant, as manifested in the derivative active forces resulting from it.

The above sketch, of course, leaves much to be explained, but it does at least indicate in general terms how Leibniz's views on substance bear on the analysis of the continuum. In order to elaborate his solution to the labyrinth in more detail it will be convenient to sketch its historical development, beginning with Leibniz's first systematic foray into natural philosophy. It is here that the relationship between the theory of substance and the composition of the continuum is most explicit.

2. *the geometry of indivisibles: the presupposition argument*

When Leibniz burst upon the intellectual scene in Europe in the early 1670s, it was as an enthusiast of the new Mechanical Philosophy. The new learned societies of London and Paris found themselves presented with a treatise on physics, *Hypothesis physica nova* ("A New Physical Hypothesis", hereafter *HPN*), sporting novel theories of a wide range of phenomena from the formation of the solar system to the cohesion of atoms, and founded on an intriguing theory of the continuum, given in an accompanying tract, the *Theoria Motus Abstracta* (*TMA*). The germ for this theory is the identification of substance—which for Leibniz is by definition indivisible—with the indivisibles of geometry. Here he builds on Hobbes's treatment, but where the Englishman had gone for an uncompromising finitist reinterpretation of indivisibles or points, Leibniz blithely

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4 *De ipsa Natura*, §13; GP IV 513.

5 "Remarques sur Foucher"; GP IV 491.

substitutes an equally uncompromising commitment to the actually infinitely small. Thus Hobbes had rejected the Euclidean definition of a point as "that which has no part" in favour of "that whose part is not considered"; Leibniz rejects both in favour of "that which *has no extension*, i.e. whose parts are indistant".[6] On the one hand, the bold assertion that indivisibles contain parts avoided Aristotle's objection (in *Physics* 231b) that indivisibles, being partless, cannot be composed into a continuum; on the other, since Leibniz defined magnitude as "the multiplicity [*multitudo*] of its parts", Leibniz's points (unlike Galileo's *parti non quante*) could have a magnitude. Their ratio to a finite line would then be as 1 to ∞, not 0 to 1,[7] as would a partless point or minimum. The latter is rejected on geometric grounds: "a minimum cannot be supposed without it following that the whole has as many minima as the part, which implies a contradiction."[8]

In justification of this idea of points with parts but no extension Leibniz followed Hobbes in appealing to horn angles, angles between a straight line and a curve, one of which could be bigger than another even while both are less than any rectilinear angle that can be assigned. He also appealed to the Scholastic doctrine of signs in support of the idea that even simultaneous things could have an order of priority, an ordering of parts without extension. But his strongest argument was an ingenious inversion of Zeno's dichotomy argument applied to motion. If motion is to occur, then it must have a beginning. But whatever is moving in a given interval must already have been moving in the first half of that interval, so the beginning of the motion must be contained in this interval. But what is moving in the first half must already have been moving in the first half of this half, and so on to infinity. Zeno concluded that motion could never begin; Leibniz (taking for granted the reality of motion) concluded that since the beginning of any motion cannot consist in an extended stretch of motion, this beginning must be unextended. Indeed, since this argument is applicable to any subinterval of the motion—or likewise to any subintervals of lines, bodies or times—it entails the stronger conclusion that any subinterval whatever must contain an unextended beginning. Thus a beginning of motion, which Leibniz followed Hobbes in calling an endeavour (*conatus*), is proportional to the beginning of a line (point) covered in the beginning of an interval of time (instant). If follows that if we take two points *p* and *q* that are the beginnings of two different lines described in time T by the unequal uniform motions whose speeds are M and

6 *Theoria Motus Abstracta* (*TMA*); A VI 2, 264-65.
7 *TMA*; A VI 2, 265.
8 *TMA*; A VI 2, 264. The argument is given below.

N, they will be proportional to the endeavours that are the beginnings of these motions, M/∞ and N/∞, resp. Therefore even though these points are infinitely small they will be in the ratio M:N, i.e. in the same ratio as their generating motions. An infinity of points of length MT/∞ will compose a line of length MT, just as an infinity of endeavours M/∞ will compose the motion M.

In the *HPN* Leibniz used this theory of indivisibles of different sizes to explain the cohesion of bodies in terms of the fusing together of their boundaries. When one body impinges on another, the indivisible constituting the boundary of the impelling body is greater than that of the body with which it is colliding in proportion to the endeavours of their motions; their boundaries therefore overlap, so that the bodies are truly continuous and cohering, as opposed to being merely contiguous. By this means Leibniz sought to explain the original cohesion of the hollow atoms or *bullae* (literally, bubbles) that he claimed in the *HPN* were formed shortly after Creation like the little globules of glass shooting off in a glassworks.[9] The spinning motion imparted by the Sun's rays would result in concentric rings of matter made to cohere by the endeavours propagating around them.

It is the connection of endeavours with indivisibles, moreover, that carries the metaphysical weight in Leibniz's early theory. Following Hobbes's construal of endeavours as incipient desires and aversions, with the mind a kind of means for containing contrary endeavours beyond a moment, Leibniz construed bodies as "momentary minds". By this means, he boasted to Oldenburg in 1671, he could "demonstrate" the indestructibility of mind from the fact that "once two contrary endeavours in the same point of a body are compatible beyond a moment, no other bodies can slip between them, nor can they be prized apart by any force for all eternity".[10] He advertised this theory to Duke Johann Friedrich as providing a firm foundation for the Christian dogma of bodily resurrection, and as being able to circumvent the problem posed by cannibalism: the idea was that the mind, "safe and sound in its point" could lie dormant until resurrection, when its sphere of influence could expand to a full-size body again, the mind determining the identity of the individual.[11] Moreover, if the mind is contained in a mathematical point at the extremity of a pointed body, a division of the body resulting in a division of the point could produce a

9 *HPN*; A VI 2, 226.
10 Letter to Oldenburg, 1671; A II 1, 90.
11 Letter to Duke Johann Friedrich, May 1671; A II 1, 113.

multiplication of minds. By this means, he claimed in a letter to Lambert van Velthuysen[12], he could use this theory of indivisibles to explain "with as much clarity as sunshine" how "mind can multiply itself, without new creation, by traduction, with no mention of incorporeality". (Traduction is the idea that in biological generation, the soul of the offspring is not created *de novo* by God, but instead results from a kind of budding of the parents' souls together with the genetic matter, by analogy with the grafting of trees.)

3. the differential calculus: infinitesimals and infinite wholes as fictions

By the time Leibniz had arrived in Paris in 1672, however, he had come to see that the distinction he had made in the *TMA* between indivisibles and minima could not be sustained. The same argument that he had given in the *TMA* against minima (partless points) could be applied to indivisibles: the points on the diagonal of a rectangle could be put into 1-1 correspondence with the points on one side, and these with the points on a part of the diagonal equal in length to the side, so that there would be as many points in the part of the diagonal as in the whole, contrary to the part-whole axiom.[13] Nevertheless, Leibniz still upheld the infinitely small beginnings he had established by the dichotomy argument, provided they were regarded instead as defined by motion. Thus two lines generated in the same time by different generating motions would have at any instant infinitely small lengths proportional to those motions. Consequently, from now on Leibniz would sharply distinguish such infinitesimals from points. Points, he realized, must be mere endpoints, the boundaries of a line segment. As such, they would be of dimension 0, and thus not homogeneous with a line of dimension 1. An infinitely small part of the line, on the other hand, must be homogeneous with the whole of which it is part; it would therefore have extension, be further divisible, and would itself be bounded by endpoints.

This conception of infinitely small parts as dimensionally homogeneous with the whole they composed was a notion Leibniz found confirmed in the mathematical work of Pascal, which he was reading at this time (winter 1672-3) under Huygens' guidance. On this conception, the area under

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12 Letter to van Velthuysen, May 1671; A II 1, 97-98.

13 This argument depends for its success on Leibniz's conception of magnitude as the "multitude of its parts". So long as points are conceived as parts of the line, the multitude of points composing the diagonal will be both greater than and equal to the points composing the side, because the magnitude of the diagonal is greater than that of the side. Leibniz held onto this definition of magnitude, but came to deny the composition out of indivisible points, i.e. that points were parts. Of course, that a proper part (i.e. subset) of a set can be put in 1-1 correspondence with the whole set is later taken by Dedekind and Cantor as the very definition of an infinite set. But Leibniz insisted that the part-whole axiom must apply without exception, and therefore denied that an infinite set could be regarded as a collection.

a curve could be regarded as made up of an infinity of infinitely small rectangular areas, each with the height of an ordinate and with an infinitesimal width, at least as a means of calculation. Also with Huygens' encouragement, Leibniz had quickly made great progress in his studies of infinite series, discovering a simple relationship between a series of terms and a second series whose terms are the differences of the first, his "Difference Principle". If, from a given series A, one forms a difference series B whose terms are the differences of the successive terms of A, the sum of the terms in the B series is simply the difference between the last and first terms of the original series: "the sum of the differences is the difference between the first term and the last".[14] Leibniz immediately applied this result to infinite series whose infinitieth term could be taken to be 0, yielding results for sums of infinite series such as that of the reciprocal triangular numbers, $^1/_1 + {}^1/_3 + {}^1/_6 + {}^1/_{10} + {}^1/_{15} + \ldots = 2$.

With his characteristic genius for generalization, Leibniz extended these considerations to continuous quantities, and within a mere three years was led to formulate what we now call the Fundamental Theorem of the Calculus: if dx is the infinitely small difference between the infinitely many successive values of x, the area under a curve $y = f(x)$ (its quadrature, Q) between $x = a$ and $x = b$ can be represented as an infinite sum: $Q = \int_n f(x_n)dx$. Now if one has an expression for the general term of another series $g(x_n)$, the difference between whose successive terms is $f(x)dx$, one may apply the Difference Principle: the sum (integral) of the differences equals the difference of the first term and the last (the definite integral evaluated between first and last terms), $\int_a^b f(x)dx = [g(b) - g(a)]$.

Some of Leibniz's reasonings on the way to formulating his calculus are of special interest with regard to its foundations. Thus in a paper written in 1674, Leibniz calculated the area (ACBEM on his figure) under the hyperbola $x = 1/(1 - y)$ between the y-axis and the line $y = 1$ by expanding it as an infinite series of terms $x = (1 - y)^{-1} = 1 + y + y^2 + y^3 + \ldots$ and then effectively integrating term-wise to obtain ACBEM = $1 + {}^1/_2 + {}^1/_3 + {}^1/_4 + {}^1/_5 + \ldots$, which is infinite. By a similar means he was also able to calculate a finite area under the curve on his diagram, CFGLB, as $1 - {}^1/_2 + {}^1/_3 - {}^1/_4 + {}^1/_5 + \ldots$. But on subtracting the finite space CFGLB from the infinite space ACBEM, he obtains ACBEM – CFGLB = $(1 - 1) + ({}^1/_2 + {}^1/_2) + ({}^1/_3 - {}^1/_3) + ({}^1/_4 + {}^1/_4) + ({}^1/_5 - {}^1/_5) + \ldots = 1 + {}^1/_2 + {}^1/_3 + {}^1/_4 + {}^1/_5 + \ldots$ = ACBEM. Thus "the area of the space [ACBEM] remains the same even

[14] A VII 3, 95.

when the finite space [CFGLB] is subtracted from it". From this he concludes that "the infinite is not a whole, but only a fiction; for otherwise the part would be equal to the whole."[15]

Here we see the emergence of Leibniz's doctrine of the fictionality of infinite magnitudes. But the connection with infinite series is particularly revealing since, in the above reasoning, treating the space ACBEM *as if* it were a whole is equivalent to treating the infinite series *as if* it had a last term or *terminatio*, $1/\infty$; indeed this is presupposed by all his applications of the Difference Principle. This gives the all-important connection between Leibniz's doctrines of the fictionality of infinite wholes and the fictionality of infinitesimals. One cannot, however ascribe a property to "the infinite as a whole, except when there is a demonstration of it" (A VI iii 168). For example, by showing that the general term of the series of reciprocal triangular numbers is $2/(y^2 + y)$, Leibniz was able to show that the sum is $2 - 2/(y + 1)$, which approaches 2 arbitrarily closely as y is taken arbitrarily large.

By 1676 Leibniz had built these beginnings into the differential calculus, creating the notation and rules for differentiation that we use today, and with explicit recognition of the inverse nature of integration (Bernoulli's term) and differentiation. He had also developed the above intuitions about the fictional character of the infinite and the infinitesimal into a sophisticated foundation for the calculus based on the Archimedean axiom, which implies that no matter how small a geometric quantity is given, a smaller can be found.[16] On this conception, an infinitesimal is a *compendium* (an abbreviated expression) for a variable finite quantity that can be made as small as desired. The infinitesimal is treated *as if* it is an actual quantity in certain defined circumstances, although one that can be ignored by comparison with a finite quantity. For example, in a formula obtained by treating dx as a non-zero quantity, the neglect of expressions involving dx in comparison with x is justified by reference to the Archimedean axiom. If $y = x + dx$, where dx is an arbitrarily small variable quantity, and D is any pre-assigned difference between y and x, no matter how small, then dx may always be taken so small that $dx < D$. Therefore, since dx, the difference between y and x, is smaller than any assignable, it is unassignable, and effectively null. Thus a converging infinite series can be treated as if it were an infinite sum with a last (infinitely small) term, and this is justified by the fact that the difference between this and the sum of a finite series with the same first term and

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15 A VII 3, N. 38_{10}, 468 [October 1674].

16 The axiom states: "Those magnitudes are said to have a ratio to one another which are capable, when multiplied, of exceeding one another" (Euclid, *Elements*, Book V, Def. 4). That is, for any two geometric quantities x and y (with $y > x$), a natural number n can be found such that $nx > y$. It also follows that no matter how small a geometric quantity x is given, a smaller (y/n) can be found.

law of progression can be made smaller than any quantity assignable, and thus null. Similarly, the area under a curve y = f(x) can be treated as if it were the sum $\int f(x)\, dx$ of rectangles of height y and width dx where the dx are infinitely small differences of (fictional) successive values of x, and this is justified by the fact that the difference between this area and the sum of the sum of a finite number of finite elements of area can be made smaller than any predetermined value. Later Leibniz would develop these ideas into his Law of Continuity: "If any continuous transition is proposed that finishes in a certain limiting case (*terminus*), then it is possible to formulate a general reasoning which includes that final limiting case."[17]

4. the actual infinite

In keeping with this account of the infinitely small, and perhaps also stimulated by Spinoza's related views on the infinite which he studied in early 1676, Leibniz articulated a subtle theory of the actual infinite. According to this view, it is perfectly legitimate to describe a plurality of things as actually infinite, without that committing you to there being an *infinite number* of them. To say that there are infinitely many prime numbers, for example, is to say that for any finite number you choose, there are actually (not merely potentially) more primes than this; it is *not* to say that there is a number of primes that is greater than any finite number, i.e. an infinite number of them. The infinite, like the infinitely small, is thus treated as a *syncategorematic* term, one that derives its meaning from the sentence in which it occurs, but one for which there is no corresponding entity.[18]

The significance of this for Leibniz's theory of matter can be seen as follows. He had long been firmly committed to the division of matter to infinity, and as we saw, initially interpreted this division as issuing in indivisibles. But with the collapse of his earlier attempt to distinguish indivisibles from minima, Leibniz was unable to see how to avoid the conclusion that an actually infinite division of matter would issue in minima. In some of the fragments he penned in Paris that last spring of 1676, he experimented with the idea that a solid body or atom would be an infinity of such material points held together by a mind, although he no longer seemed sure how the mind could achieve this. Finally, as his calculus took shape, he came to see that infinite division need not issue in points after all. If matter is infinitely divided in the syncategorematic sense, each part will be

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17 *Historia et Origo calculi differentialis a G. G. Leibnitzio conscripta*, (ed. C. I. Gerhardt), Hanover, 1846.

18 "It is easy to demonstrate that there is no infinite number, nor any infinite line or other infinite quantity, if these are taken as genuine wholes. The Scholastics were taking that view, or should have been, when they allowed a *syncategorematic infinite*, as they called it, but not a *categorematic* one." *Nouveaux Essais,* xvii, A VI 6, 157.

divided in such a way that, no matter how small a part is taken, it is divided further still. He found support for this in the argument in Descartes' *Principles* (or perhaps Spinoza's rendition thereof) that bodies are actually divided by the differing motions of the parts inside them. For a part of matter can be individuated by all its constituents having a motion in common, without this precluding these constituents having other motions which further divide the part. No reason could be given, he concluded, for God's "putting a stop to his handiwork" and ceasing this division at any point.[19] There is therefore no such thing as a perfect liquid, one consisting in points: "on the contrary, every liquid has some tenacity, so that although it is torn into parts, not all the parts of the parts are so torn in their turn; instead they merely take shape for some time, and are transformed; and yet in this way there is no dissolution all the way down into points, even though any point is distinguished from any other by motion."[20]

In the late 1670s and 1680s Leibniz builds on this conception of the actual infinite division of matter to argue that body, understood as something merely material, has no more than a phenomenal unity. Taking as a premise the nominalist principle that *the reality of an aggregate derives only from the reality of its constituents*, he argues that body will have "no reality at all if each constituent being is still a being by aggregation, for whose reality we have to find some further basis".[21] If a body is the aggregate of the parts into which it is divided, then its reality consists in the parts alone, and not in their being perceived as one. But since each of these parts is further divided, the argument iterates: the body is a perceived unity and a plurality of parts, but each of these parts is also a perceived unity and a plurality of parts, and so on down. If there are no true unities, then, given infinite division, the reality of body will elude analysis, and it will exist merely as a perceived unity, a pure phenomenon. If, on the other hand, there exist true unities in the body, then the body's reality will reduce to the reality of these, while its own unity will consist only in their being perceived together. It will then be what Leibniz in his correspondence with De Volder calls a "quasi-substance", a plurality of substances with no substantial unity. This returns us to a consideration of substance.

5. *atoms of substance and the plurality of forms*

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19 *Pacidius Philalethi* [1676]; A VI 3, 561.

20 *Pacidius Philalethi*; A VI 3, 555.

21 Letter to Arnauld, 30th April 1687; GP II 96.

Although by the Spring of 1676 Leibniz had abandoned the composition of the continuum from indivisibles, and had arrived at his interpretation of geometric infinitesimals as fictional parts of the line, he was still not sure what the implications were for his theory of substance. He had strong motivations for retaining mind-containing atoms of some kind, since, as we saw, he thought he could explain the immortality of rational souls in terms of their being contained in indivisibles, and thereby also explain the propagation of souls by traduction. Also, as we have seen, his earlier theory was motivated in part by the idea that the mechanical philosophy was incomplete without some principle of activity from which the motions of bodies would derive, and he had seized on Hobbes's endeavour as providing such a principle. Thus as late as April 1676 Leibniz was entertaining the idea that every atom contains a mind or soul as its principle of individuation, each associated with its own vortex, and that "the soul itself activates this vortex."[22]

In the summer of 1676, however, Leibniz underwent a kind of conversion from the materialist trend of these speculations. This may have been induced by his reading of Plato's dialogues, two of which he was then translating, particularly the passage from Plato's *Phaedo* where Socrates criticizes his former teacher Anaxagoras for "introducing mind but making no use of it".[23] The idea is that Socrates' remaining on the bench in prison cannot be understood in terms of material causes, and can only be understood teleologically, i.e. in terms of his mind acting according to its own end-directed laws, in step with his body acting according to the laws of mechanics. Any model of the mind such as Leibniz had previously been entertaining, where the mind acts on the matter of its own vortex, would from now on be rejected out of hand as too naive. Cementing him in this opinion would have been the Occasionalist arguments familiar to him from his discussions with Malebranche in his four years in Paris. Malebranche argued that the purely passive nature of body—conceived as mere extended substance—precludes it from having any power to "transmit to another body the power transporting it",[24] a power which must therefore reside only in God. Although Leibniz wanted each body to contain its own principle of activity, and thus to be more than mere extended substance, he had to concede to Malebranche that there was no way to conceive how such a principle could act on anything outside of itself.

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22 *De unione animae et corporis* [February(?) 1676], A VI 3, 480.

23 Plato, *Phaedo,* 97b-99c. In his *Discours de métaphysique* of 1686, Leibniz makes a marginal note to himself to insert the translation of this passage at §20, "A memorable passage by Socrates in Plato's *Phaedo*, against over-materialistic philosophers" (GP IV 446; GP II 13).

24 Nicolas Malebranche, *The Search after Truth* (trans. T. M. Lennon and P. J. Olscamp. Columbus: Ohio State University Press, 1980), p. 660.

Leibniz was, nevertheless, firmly committed to the mechanical philosophy. As he affirmed to Hermann Conring in January 1678, he regarded as "demonstrated" Robert Boyle's claim "that *everything in nature happens mechanically*."[25] When Conring professed alarm at this apparent endorsement of Cartesianism, Leibniz reassured him that he was very far from rejecting the substantial forms necessary for individuating parts of matter.[26] Boyle himself, of course, did not regard mechanism as incompatible with an inherent teleology in created things mandated by God's providence. But he vehemently rejected substantial forms, attaching to the first edition of his *Origin of Forms* a lengthy critique of them. His main target was Daniel Sennert, who in 1636 had recommended against the atomism of his contemporary Sebastien Basson that substantial forms should be retained.[27] Atoms must be animated, Sennert had claimed, in order to account for such phenomena as the formation of minerals and biological generation. Now Leibniz agreed with Boyle about not employing forms to explain particular phenomena, but he could not agree that matter left to itself would have any means for self-organization, nor with Boyle's conception of teleology as simply imposed from above by divine fiat. On these points, therefore, Leibniz's sympathies would have lain with Sennert rather than Boyle. For Leibniz, as for Sennert and other Lutherans such as de Goodt, God's instruction in *Genesis* 1: 22, "Be fruitful and multiply!", could only be understood to mean that the entities created by God in the first days contained within them the capacity for propagating all their descendants. They must possess, in Leibniz's words, "a form or force … from which the series of phenomena will follow according to the dictate of the original command".[28]

Consequently Leibniz appealed to the same tradition of Latin pluralism as had Sennert, advocated above all by J. C. Scaliger, in trying to resolve the problem of forms. According to this tradition, matter, prior to its being organized by a form into an organic body, does not exist as a pure potentiality (as in the orthodox Thomist tradition), but instead has its own reality. This reality has its source in the parts of a body themselves being made actual by their own subordinate forms. Boyle's sarcastic criticisms about how these mysterious forms could arise or perish are then neatly side-stepped by Leibniz's proposal "that every soul, and indeed every corporeal substance, has

25 Herman Conring in January 1678.
26 Herman Conring in January 1678.
27 Daniel Sennert, *Hypomnemata Physica* (Frankfurt, 1636).
28 *De ipsa Natura*, §6; GP IV 507.

existed from the beginning of things".[29] Thus the forms or forces responsible for the teleology of bodies are coeval with the created world, and they are identified by Leibniz with the true unities presupposed by every existing body, as demanded by the infinite aggregate argument described above. Every actual part of matter is either the organic body of a substance (like an animal) or a collection of such (like a woodpile): "all bodies are either organic or collections of organic bodies".[30] The organic body is animated by a dominant form, but the parts of the body are actualized by their own subordinate forms: every organism contains within its body further subordinate organisms, and "every generation of an animal is only a transformation of an already living animal"[31]—a conception for which Leibniz saw significant empirical support in the microscopic observations of Leeuwenhoek and Malpighi. But how do these considerations about substance relate to the continuum?

6. substance as force

According to Leibniz, the Ariadne's thread that led him out of the labyrinth was the estimation of forces. At first blush it appears as though he must here be referring to a different labyrinth, since it is not at all clear how a specific issue in physics—namely, the correction of the Cartesian measure of force, mv ("quantity of motion"), to the Leibnizian measure, mv^2 ("living force")—could be relevant to the problem of the continuum. There is, however, a deep connection through the definition of substance as *something that acts*, a definition that Leibniz takes as axiomatic. His upholding of this idea of substance is in itself an implicit criticism of Cartesianism. There must be a "this something" (Aristotle), and there must be some measure of the activity of this something at each instant. To begin with, as we saw above, Leibniz saw a solution in the Hobbesian notion of endeavour, identifying this as the principle of activity of bodies. But, as he recounts in many writings, if a body consisted only in size, shape, position and their changes, together with such an endeavour, this would have the unwelcome consequence that "the largest body at rest would be carried away by the smallest body colliding with it".[32]

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29 From *Mira de natura substantiae corporeae*, [March 1683]; A VI 4, 1466; LoC 265). Cf. "Whatever acts cannot be destroyed, for in any case it endures as long as it acts, and will therefore endure forever" (*De origine rerum ex formis* [April 1676]; A VI 3, 521).

30 *Ex Cordemoii tractatu*; A VI 4, 1798.

31 Draft of a letter to Arnauld, 8 December 1686; GP II 72.

32 *Specimen Dynamicum*, GM VI 241.

This is where Leibniz's revamped notion of substance comes into play. The argument so far has been that there must be real unities in matter, and also that there must be some principles by which the differing motions in matter at any instant might be distinguished. These desiderata are both satisfied by Leibniz's conception of the unities or substances as beings capable of action, for which it is necessary for them to be repositories of *force*. On the one hand, force is "an attribute from which change follows, whose subject is substance itself"[33]; on the other, it involves an endeavour or striving, and this is what the reality of motion consists in: "there is nothing real in motion but that momentaneous thing which must be constituted by a force striving towards change".[34] This force is thus the foundation for the motion of any actual part of matter at any instant. It is an *entelechy* in the sense that it remains self-identical through the changes of state that it brings into actuality: it is the real foundation at any instant for the momentary state of motion individuating the actual part of matter that constitutes its organic body, as well as for the derivative passive forces of impenetrability and resistance. Derivative force is momentaneous, and since "what is momentary in action … is accidental or changeable",[35] it exists only as a modification of the primitive force that it presupposes.[36] The differing tendencies to motion to which the entelechies in matter give rise are what make actual the various parts into which matter is divided at different instants. Infinitely divided matter, that is, presupposes entelechies which determine its actual parts.

This bears on Leibniz's philosophy of space as follows. According to the account of the continuum that he had developed in the 1670s, space cannot be regarded as composed of points, or time of instants, without contradiction. Any finite line segment is bounded by two points, which are its modes. Between these bounds there is a continuous extension, and this may be regarded as divisible into further such segments. That is, in the abstract, a space or extension may be regarded as composed of such parts, but being continuous, it is not actually divided into them. Actual division pertains to actual entities, like extended matter, the actually divided phenomenal plenum. But extension in itself is essentially passive; in order for matter to be actually divided there must be active entities in it, the entelechies responsible for the instantaneous motions determining the different divisions it undergoes at different instants. "The extended," Leibniz explains to De Volder

33 Letter to De Volder, 3 April 1699; GP II 170.

34 *Specimen Dynamicum*, GM VI 235. In Leibniz's dynamics, these primitive forces give rise to the derivative forces that manifest themselves in phenomena, the most important of which is the living force, mv^2, which is a measure of the activity of a body at an instant or its capacity to do work, anticipating the modern notions of energy and work.

35 Letter to De Volder, 30 June 1704; GP II 270.

36 Untitled ms. [1702], GM VI 102-3.

in 1699, "has no unity except in the abstract, namely when we divert the mind from the internal motion of the parts by which each and every part of matter is, in turn, actually subdivided into different parts, something that plenitude does not prevent. Nor do the parts of matter differ only modally if they are divided by souls and entelechies, which always persist."[37] Now every actual part of matter has a situation relative to other bodies in the plenum, and space is the order of these situations (a kind of complex of divisions or cells); abstract space consists in the order of all possible such situations. Such a space, conceived apart from the things in it that would induce the divisions, is therefore purely ideal, consisting only in the possibility for division. It is simply an order, the order of all possible situations, and not something (*contra* Newton) that could actually exist.

7. monads and the continuum

We have seen that Leibniz had initially conceived his indivisibles as actually infinitely small parts of the continuum, but that this conception did not survive his rejection of geometric indivisibles in 1671-72 and his subsequent reconstrual (in 1676) of the infinitely small parts of the continuum as fictions. This went along with a reinterpretation of the actually infinite division of matter as not issuing in a last or smallest part, but as a division only into finite parts that are themselves divided into finite parts *in infinitum*. Nonetheless, as we have seen, one feature of the earlier conception remains fundamental for his mature philosophy: this is the idea that every part of an actually divided body *presupposes* an indivisible substance that is active, and whose instantaneous action is the foundation of the body's motion. These substances *are in* matter everywhere: "Since monads or principles of substantial unity are everywhere in matter, it follows from this that there is also an actual infinity, since there is no part, or part of a part, which does not contain monads."[38]

Thus for an entity *B* to *presuppose* entities *A* is for the *A* to *be in B*. As Leibniz writes in 1714, "We say that an entity is in [*inesse*] or is an ingredient of something, if, when we posit the latter, we must also be understood, by this very fact and immediately, without the necessity of any inference, to have posited the former as well."[39] This is the way, then, that a line can be regarded as an infinite aggregate of points, and a body as an infinite aggregate of substances: "just as there is no portion of a line in which there are not infinitely many points, so there is no portion of matter which does not contain an infinity of substances. But just as a point is not a part of a line, but a line

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37 Leibniz to De Volder, April 1699; GP II 170.

38 Letter to Des Bosses, 14 Feb 1706; GP II 301.

39 *Initia rerum mathematicarum metaphysica* [1714], GM VII 19.

in which there is a point is such a part, so also a soul is not a part of matter, but a body in which there is a soul is such a part of matter."[40] Although points are not parts of a line, they are presupposed in any of its parts; analogously, monads are not parts of matter, but presupposed in any of its actual parts. Moreover, being simple, they are not further resolvable, and are therefore said to be its *constitutive principles*. As Leibniz writes to Fardella:

> There are infinite simple substances or created things in any particle of matter; and matter is composed from these, not as from parts, but as from constitutive principles or immediate requisites, just as points enter into the essence of a continuum and yet not as parts, for nothing is a part unless it is homogeneous with a whole, but substance is not homogeneous with matter or body any more than a point is with a line.[41]

As explained above, the *reality* of an aggregate reduces to the reality of its constituents, and the *unity* of the aggregate results from these constituents being perceived together.

Monads are also presupposed by temporal phenomena, but here different considerations must apply. For the constituents of a temporally extended phenomenon cannot be perceived together at the same time. There is therefore no *unity* of temporal extended phenomena. The *reality* of a phenomenal duration, on the other hand, consists in the monadic states it presupposes, and the reality of motion in the momentaneous endeavour towards change, or appetition, existing at every instant. Likewise, the momentaneous derivative forces, being modifications of primitive forces, presuppose the existence of those enduring unities or sources of action which they modify. Here it may be objected that if monadic states are instantaneous, and according to Leibniz nothing that exists only for an instant can be said to exist, his monadology does not in the end escape the labyrinth. To that objection I think the best answer is this: a monadic state is a kind of ideal limit of making finer and finer discriminations of states, each of which involves a certain degree of abstraction or limitation.[42] Thus all change presupposes a difference between two states. A continuous process is analyzable as an infinite series of infinitesimal such differences; but according to Leibniz's analysis of continuity, this is a *compendium* for the fact that finer and finer

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40 Leibniz, Notes on discussions with Michel Angelo Fardella [1690], A VI 4, 1673.

41 Leibniz, for Fardella [1690], A VI 4, 1673.

42 Cf. the illuminating analysis given by Donald Rutherford, "Leibniz on Infinitesimals and the Reality of Force", in *Infinitesimal Differences*, ed. Ursula Goldenbaum and Douglas Jesseph (Berlin: de Gruyter 2008), 255-280.

determinations of discrete states are possible, in such a way that no smallest difference (corresponding to a discontinuous change) would ever be reached.