"Beeckman's discrete moments and Descartes' disdain"

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Abstract:

Descartes' allusions (in the *Meditations* and the *Principles*) to the individual *moments* of duration has for some years stirred controversy over whether this commits him to a kind of time atomism. In this paper I trace the origins of Descartes' way of treating moments as least intervals of duration to his early collaboration with Isaac Beeckman. Where Beeckman (in 1618) conceived of moments as (mathematically divisible) physical indivisibles, corresponding to the durations of uniform motions between successive impacts on a body by microscopic particles, Descartes was able to give a mathematical treatment of the problem of fall in which moments were rendered mathematical minima of motion that were necessarily devoid of extension. This achievement, coupled with his innovation of conceiving force as instantaneous tendency to motion, subsequently led him to disdain Beeckman's discretist physics with its extended indivisible moments. Nevertheless, I argue, he was not able to eradicate a fundamental tension in his philosophy between force (as a quantity of motion), and force as an instantaneous tendency to motion. For by his principles, action, motion, quantity of motion, and indeed existence, all require some minimal interval of duration. This explains his need to refer to moments as least conceivable parts of duration, and this is what has given rise to the impression that he supposed duration to be composed of such parts, contrary to his commitment to continuous creation.

1. Introduction

Much of the recent discussion concerning whether Descartes espoused temporal atomism has proceeded by an analysis of textual evidence.[1] The chief locus of such discussion has been the passage in the Third Meditation:

> I do not escape the force of these arguments by supposing that I have always existed as I do now, as if it followed from this that there was no need to look for any author of my existence. For since every lifetime can be divided into innumerable parts, each of which in no way depends on the others, it does not follow from my having existed a short while ago that I must exist now, unless there is some cause which creates me as it were again at this moment—that is, conserves me. For it is quite clear to anyone who attentively considers the nature of time that the same force and action is plainly needed to conserve any thing at each moment it endures as would be needed to create it anew if it did not yet exist. Thus the fact that the distinction between conservation and creation is only one of reason is also one of those things made manifest by the natural light.[2]

Commentators who have seen in this a profession of time atomism have taken the innumerable independent parts into which a duration can be divided to be the moments through which each thing endures, and thus to signal a composition of duration out of innumerable discrete parts. Of course, as several scholars have observed, there was ample historical precedent for Descartes, in his own epoch, to have subscribed to temporal atomism of some stripe.[3] There was the time atomism of the Mûtakallimûn (proponents of *kalam*), which Maimonides had made known to the West, according to which each substance lasted no longer than one time atom, and then had to be created anew. Inspired by the Mûtakallimûn, perhaps, were the views of Nicholas of Autrecourt and Gerard of Odon, and in Descartes' immediate intellectual

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[1] See, for example, Beyssade 1979, Gueroult 1984, Arthur 1988, Garber 1992, Secada 1990, Frankfurt 1999, Bonnen and Flage 2000, Gorham 2004, 2007, 2008, and Levy 2005.

[2] Descartes, *Meditations* 3: AT VII 48-49; CSM II 33. All translations of primary sources here are my own; but I also reference the standard English translations.

[3] See in particular Secada 1990, and Sorabji 1983.

environment, the Jesuit followers of Zeno in Leuven and Spain, the Zenonists, who advocated the composition of the continuum from points.[4]

Of course, such an atomist interpretation is by no means forced on us by the wording in the above passage, since it is possible to read Descartes as arguing for innumerable, independent *finite* parts of such a duration, with the moments or instants being merely boundaries of these temporal intervals. The existence of precedents in the views of the Mûtakallimûn or the Zenonists establishes a bare possibility for Descartes to have advocated a time atomism (as argued in Secada 1990); but what needs to be shown in addition is that he would have had the motivation to adopt such a position.

Now without going into all the detailed arguments on either side, I maintain that the prospects for showing this are very slim indeed. In the first place, there is Descartes' own hostility to indivisibles in physics. As he wrote to Mersenne in criticism of the views of Father J. Lacombe, body "has length and breadth and extension, and so cannot be composed of indivisible things, since an indivisible thing cannot have length or breadth or depth" (AT III 213; CSMK 155).[5] Similarly, he tells Henry More in 1649 that

> It would imply a contradiction for there to be atoms which are conceived of as extended and at the same time indivisible. (to More, 5th Feb. 1649; AT V 273; CSM-K 363)

In keeping with this, is Descartes' well-known opposition to the method of indivisibles in mathematics, and his concomitant refusal to become embroiled in questions concerning the composition of the continuum. To his confidante Mersenne, for example, he made various derogatory remarks about Galileo's idea of continuous matter being composed of indivisibles. The latter's proof that a straight line "is composed of an infinity of actual points" is dismissed by him as "only an imagination pure and simple" (to Mersenne, 11th Oct. 1638; AT II 383).

[4] I am setting aside the case of Diodorus Cronus, whose time atomism has only recently been established by the fine work of Richard Sorabji (1983).

[5] I am indebted to Geoffrey Gorham for reminding me of this passage.

Contrary to Dan Garber and others, I do not believe this implies an agnosticism about time atoms on Descartes' part.[6] I take the import of these objections to be good grounds for thinking that Descartes would *not* have aligned himself with the early Islamic thinkers in asserting time atoms, since duration is certainly analogous to extension in all pertinent respects. Extended moments would not be indivisible, and duration could no more be composed of durationless instants than extension could be composed of points, contrary to the claims of the Islamic atomists, Nicholas and Gerard, and the Zenonists. As I previously concluded from this line of argument, for Descartes "indivisibles and the actually infinitely small are not distinctly conceivable" (1988, 374). Nevertheless, as I have urged, the fact that Descartes was not interested in trying to solve the problem of the composition of the continuum gives us no reason to question his being committed to duration's being continuous, and there being instants everywhere in it.

A second objection to attributing a time atomism to Descartes, which in my opinion is decisive, stems from Descartes' assertions that "we should regard the duration of a thing simply as a mode under which we conceive the thing insofar as it continues to exist" (AT VIIIA 26; CSM 211), and that "since a substance cannot cease to endure without also ceasing to be, the distinction between a substance and its duration is merely one of reason" (*Principles*, I, §62: "What is meant by a distinction of reason"; AT VIIIA 30; CSM 1 214). This makes it impossible to attribute to him a view where a substance continues in existence by being literally re-created in successive moments unless these moments are contiguous, as I have argued previously against Jean Wahl's and Norman Kemp Smith's interpretations.[7] Geoffrey Gorham (2008) has used the same argument very convincingly against the interpretation of Ken Levy (2005), who ascribes to Descartes a "strong discontinuity" in time, i.e. instants separated by temporal gaps. As Gorham

[6] See Garber 1992, 273: "I will show that the issue of the continuity of time is irrelevant to Descartes' arguments". In fact, though, Garber argues this a few pages earlier in the book, concluding that "there is no strong reason for attributing *either* view [sc. temporal atomism or its antithesis] to Descartes." (269)

[7] See Arthur 1989, 353-55. As I also argue there (363-367), the supposition that parts of duration could be merely contiguous without being continuous also seems incompatible with Cartesian principles. Just as the lack of a real distinction between body and extension makes it impossible, when two bodies relatively at rest are adjacent with one another, for there to be two distinct endpoints of the bodies at one point of extension, so there would be no real distinction between the endpoints of contiguous parts of a thing's duration, so that all such contiguous parts would form a continuous stretch.

points out (2010, 15), this is simply not possible for concrete time, the "duration of the enduring thing", which is what Descartes' argument in the *Third Meditation* and *Principles* depends upon.[8] The case is therefore analogous with extension, and, as Gorham argues, we can adapt Descartes' stricture on spatial atoms to atoms of duration: "It is a manifest contradiction for time atoms to be apart, or have a gap between them, when the gap in question is nothing."[9]

Thirdly, I believe, those pointing to the antecedents of temporal atomism need to show what motivation Descartes would have had to adopt this view. Here it is often uncritically assumed that continuous creation, occasionalism and temporal atomism come as a package. Richard Sorabji gives a nice encapsulation of this reasoning:

> [In Maimonides' account of the philosophy of Ash'ari and his followers, the Mûtakallimûn] occasionalism is presented as a corollary of a further doctrine. For the Ash'arites held that every time-atom God creates an entirely new set of accidental properties … . Maimonides reports further corollaries of this view: things do not have essential natures of their own, since it is God who creates all their properties. Again, things have no tendency of their own to persist—it is this which most of all would encourage a sense of precariousness. The doctrine that our continuation requires that God re-create us from moment to moment is repeated in Descartes, and from there it influenced the seventeenth-century occasionalists. The best known of them, Malebranche, agreed that all causation involves creation… (Sorabji 1983, 297-8)[10]

[8] In his reply to Gassendi, Descartes writes: "And this is clearly demonstrated by what I explained about the independence of the parts of time, which you try in vain to evade by proposing 'the necessity of the sequence which exists among the parts of time' *considered in the abstract*. It is not this that is at issue here, but rather the time or duration of the enduring thing, and you will not deny that the individual moments of this time could be separated from those next to them, that is, that the enduring thing could at any single moment cease to exist." (AT VII 369-70; my translation; CSM II 254-55).

[9] Gorham 2008, 635; cf. Descartes' "It is a manifest contradiction for them [bodies] to be apart, or to have a distance between them, when the distance in question is nothing' (AT VIIIA: 51, CSM 1 231).

[10] In his discussion of Augustine's and Malebranche's shared doctrine that continuance of any thing in existence depends on God, Sorabji also attributes the crucial difference between Malebranche's occasionalism and Augustine's non-occasionalist view to the fact that "what [Malebranche] adds, and what is missing, so far as I know, from Augustine, is the idea that continuation depends on continuous re-creation" (304). Continuous re-creation is continuous creation interpreted as involving discrete time, so again we see continuous creation, occasionalism and temporal atomism understood as a package.

Sorabji is suggesting at least a nascent occasionalism[11] in Descartes' thought, as evidenced by the near-unanimous occasionalism of most of his immediate followers—e.g. Louis de La Forge, Géraud de Cordemoy, Nicolas Malebranche, Arnold Geulincx, and Claude Clerselier,[12]—and he is also assuming that such an occasionalism implicitly involves time atomism. But recent scholarship has established (at least, to my satisfaction) that Descartes is not an occasionalist. He does indeed commit himself to the doctrine that the continuous action of God is required to keep creatures in being from moment to moment, but he neither commits himself to the view that God thereby creates all their accidents anew at each moment, nor does he deny that minds are causally active.[13] Moreover, as I have argued elsewhere (Arthur 2012), there is no such mutual implication as is often assumed between occasionalism and time atomism. Time atomism existed in the earliest stages of Islamic atomism, perhaps as a development of the views of Diodorus Cronus or Epicurus, perhaps influenced by Buddhist atomism;[14] the Ash'arite philosophy reported by Maimonides, in which all causation is reposited in God alone (—certainly not a Buddhist precept!—) is a later development. So too is al-Ghazâlî's critique of essences or natures, which undermined the idea of a necessary connection between cause and effect.[15] In a similar vein, Louis de La Forge derives his occasionalism from his own understanding of the doctrine of divine sustenance as God's being "causally responsible at each instant not just for the existence of the body, but also for its modes or properties" (Nadler

[11] I take occasionalism, for the purposes of this essay, to be the doctrine that all real causation resides with God, and that what we uncritically call causes among created things are mere occasions for God to cause the changes in question.

[12] Cf. Sukjae Lee's comment: "Given how many prominent Cartesians were indeed occasionalists, it is not surprising that Descartes has been suggested as the source of this new wave of occasionalism that swept through the European continent in the second half of the seventeenth century." (Lee 2008). But, in fact, many qualifications have to be made. As Nadler has argued persuasively, La Forge was "an occasionalist with respect to bodies, to matter in motion", but "*not* an occasionalist when it comes to the mind" (1998, 224); similarly Clerselier. But I will not dwell on such niceties here, since all these thinkers denied body-body causation, which I think is sufficient to distinguish them from Descartes.

[13] For this conclusion, see especially the arguments of Garber 1986, 1992, 1993, and Nadler 1993b, 1998, 2005.

[14] See Sorabji's brilliant discussion in his 1999, chapter 24: Atoms and Time-Atoms after Aristotle, 365-383, and ch. 25, Atoms and Divisible Leaps in Islamic Thought, 384-402.

[15] Here I have relied on M. Fakhry 1958. Although al-Nazzam held that substances and their accidents revert back to nothingness before being recreated each moment, the attribution of this view to the Mûtakallimûn has been disputed by Alnoor Dhanani (1994, 43-47). See Nadler 1998, n. 17, 217.

1998, 219), pre-eminently its place relative to other bodies. Thus a body will trace a given trajectory if and only if God creates it at every point of the trajectory at the corresponding instants of time. This follows whether time is discrete or continuous, even though La Forge himself understood the moments to be discrete and consecutive.

This lack of a mutual implication between occasionalism and time atomism should perhaps already have been obvious from the case of Malebranche, perhaps the most famous and influential of the Cartesians, who saw no incompatibility between his ardent espousal of occasionalism and an insistence on continuous creation and the continuity of time. He argued as follows:

> On the part of God, the conservation of creatures is simply their continued creation. I say, on the part of God who acts. For, from the perspective of creatures, there appears to be a difference, since, in creation, they pass from nothing to being whereas, in conservation, they continue to be. But, in reality, creation does not pass away because, in God, conservation and creation are one and the same volition, which consequently is necessarily followed by the same effects.[16]

But a comparison of Descartes with St. Augustine is even more to the point, since Augustine upholds continuous creation and its equivalence to conservation without a hint of either time atomism or occasionalism:

> If the Creator's virtue were at any time to be missing from the created things which are to be governed, at once their species would go missing, and the whole of nature would collapse. For it is not like the case of a builder of houses who goes away once he has built, but whose work stands, even though he is missing and goes away. The world could not last like this for the duration of an eyeblink if God were to withdraw his governance from it.[17]

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[16] Again, see Nadler's excellent discussion in his 1998.

[17] Augustine, *de. Gen. ad Lit.* 4.12.22; quoted in translation from Sorabji 304. The same doctrine is repeated by St. Thomas Aquinas: "Creatures are conserved in being by God … . For the being of each creature depends on God in such a way that, unless creatures are conserved in being by the operation of the divine power, they could not subsist for a moment but would be reduced to nothing [*Creaturae conservantur in esse a Deo … . dependet enim esse cujuslibet creaturae a Deo ita quod nec ad momentum subsistere possent sed in nihilum redigerentur, nisi operatione divinae virtutus conservantur in esse*]." (*Summa Theologiae,* Ia., q. l o 4, art. 1, responsio; quoted from Nadler 1998, 216).

There are perhaps echoes of this in the passage from the *Third Meditation* quoted above. But whether or not that was the intended allusion there, it (or Aquinas's version of the same doctrine) certainly was the intended allusion of the reply Descartes made to Gassendi's criticisms of that passage:

> When you deny that *we continually need the influence* [literally, "inflow"] *of the first cause for our conservation*, you are denying a thing which all the Metaphysicians affirm as self-evident, but which the uneducated often fail to think of, because they attend only to causes of *coming to be* [*secundum fieri*], but not to those of *being* [*secundum esse*]. Thus an architect is the cause of a house and the father of his child only in the sense of being the causes of their coming into being; and hence, once the work is completed it can remain in existence quite apart from the cause in this sense. But the sun is the cause of the light which it emits, and God is the cause of created things, not only *secundum fieri*, but also *secundum esse*, and so he must always influence [*literally*, "flow into"] the effect in the same way in order for it to be conserved. (AT VII 369-70; CSM II 254-55; Arthur 1988, 362)

This echo of Augustine in Descartes is significant because Augustine is not himself an occasionalist, even though he restricts all creative power so that it ultimately resides in God (Sorabji 1983, 302-3). This is because he does not deny that created souls have within them the power to act, but insists that in doing so "they do not create, but only make use of the forces supplied by God to bring forth what he has already created" (302). Augustine locates these forces in the *seminal reasons* in created beings, provided by God at Creation, a doctrine deriving from and synthesizing prior doctrines of Stoic and Neoplatonic origin. This device allows him to make room for secondary causes without denying that all such causes depend completely on God. There is thus nothing in the doctrine of continuous creation and its equivalence to conservation in being (usually abbreviated CCC) that automatically entails either occasionalism or time atomism.[18] If God conserves a creature in being (*in esse*) by continuously creating it, he will be keeping in existence its seminal reason (according to Augustine) or substantial form (according to Aquinas), from the actions of either of which its accidents may

[18] I therefore dissent from Sukjae Lee's conclusion of his SEP article on occasionalism that "If we bracket the issue of how to render a full-blown occasionalism with a sufficiently robust account of free will, then, as we have hopefully seen, the CCC argument is a fairly powerful argument as an argument for global occasionalism." (Lee 2008).

flow. Similarly with Descartes: if divine concurrence is necessary to conserve each body in existence, and there would be no resulting force without this divine action, it does *not* follow that a body that is conserved in existence by divine action does not have a force or is incapable of causal action.[19] The body will have the motive force that it does provided God sustains the world by his action, but without this action, the world and all the bodies in it will have no force and no continued existence.

In sum, since arguments for the thesis that Descartes subscribed to occasionalism derive only from his subscribing to CCC, this parallel with Augustine is certainly strong enough to defeat the attribution: there is no convincing evidence that Descartes was an occasionalist, or that he denied bodies the ability to move one another by collisions.[20] Moreover, if Augustine and Aquinas could uphold continuous creation without this entailing a commitment to atoms of duration, the arguments on this basis for Descartes' being committed to them are similarly vitiated.[21]

Nevertheless, I am also persuaded that there are aspects of Descartes' treatment of time that are suggestive of atomism, even if that was not his intention. For *something* has to explain how it came to be that by the time of Bayle and Leibniz, the interpretation of continuous creation as *dis*continuous *re*creation was the standard view imputed to the Cartesians. This begins with Louis de La Forge, one of the originators of occasionalism, who we will take as our representative Cartesian. It is the huge influence of his and Cordemoy's arguments (also suggested by Geulincx, and extended and propagated forcefully by Malebranche), that accounts

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[19] For a defence of the view that Descartes really ascribes a force of a body's motion to a moving body, see my 2007. I think force is in body only so long as God sustains it by his creative action. I am not persuaded by Dan Garber's arguments in his 1992 that "Cartesian force is nowhere at all" (297).

[20] I do not mean to deny that for Descartes God is the ultimate cause of everything in the universe that does not depend on human free will. As he writes to Elizabeth, "he would not be supremely perfect if anything could happen in the world without coming entirely from him" (AT IV 314; CSMK 272). But this does not entail that he is the immediate cause, as in occasionalism. On my reading, bodies cause things to happen provided they are sustained in existence and in motion by God. For a different reading, see Gorham 2004, esp. pp. 410 ff.

[21] Compare with Geoffrey Gorham's argument that the independence of the parts of duration for Descartes is a consequence of their causal non-simultaneity, not their discreteness: "Descartes derives the key premise of his argument—the independence of the parts of time—from the very simple assumption that causes and effects are necessarily simultaneous" (Gorham 2004, 390).

for the Cartesian orthodoxy that bodies are causally inert, and therefore contain no action at all. La Forge argued that the action of one body on another was unintelligible on Cartesian principles. Since a body is a portion of extension and nothing else, it is passive and inert, and contains no principle for continuing its own motion, nor, a fortiori, for causing motion in any other body. Motive force, therefore, is not inherent in the body. Force of a body's motion, therefore, must be simply the will of God, and its motion consists in its being re-created by God at different relative places in different moments. This contrasts with Descartes' own view, according to which a body has just as much force and action at one moment as it did at a previous moment, provided no other cause has acted on it, and provided God continually sustains it in existence from the prior moment to the latter.

There is a difficulty with Descartes' conception, however. Given that substance or existence on the one hand, and duration on the other, are only conceptually distinct, a body cannot exist unless it endures. But an instant is a privation of duration. Therefore, in order for God to conserve something in existence, it must endure for longer than an instant. So far, so good, one might say; naturally, conservation requires a continuous stretch of time, and therefore God's continuous action throughout that stretch, i.e. at every single instant of it. Thus a body continues to exist as long as God exerts his conserving action on it, and it can then be said to exist and to have force and action at every moment of this duration. Strictly speaking, though, a body can only endure, move, and have a force and action that is proportional to a motion, over a genuine (non-zero) interval of duration. So when Descartes says in the *Meditations* that "the same force and action is plainly needed to conserve any thing at each moment it endures", this suggests that these moments must be genuine intervals of duration, not instants.

Such an interpretation, at any rate, would give one explanation of how Descartes' early followers were led to posit the discreteness of time. La Forge, for example, in his sincere attempts to remain faithful to Cartesian principles, might have been swayed by such considerations in interpreting Descartes' moments as discrete intervals of duration. If the moments are discrete, then God's action consists in re-creating bodies in different places at successive discrete moments. Descartes' appeal to the nature of time is interpreted as his

imagining it divided into moments that are small intervals of duration, perhaps smallest conceivable intervals of duration during which God could be exerting his force and action. The claims that existence in each of these is entirely distinct, and that there is no cause for continuance beyond any such moment, is interpreted as the claim that duration is made up of a succession of discrete moments.

The discreteness of duration and therefore motion, though, is deeply problematic for Descartes' physics, and runs counter not just to some specific features of that physics, but to the original insights that drive it at its foundations. I will be detailing some of these below, but for now we may take as representative what Descartes says in his *Principles* about "the operation by which God conserves motion in matter":

> For he always conserves it in the precise form in which it is occurring at the very moment when he conserves it, without taking any account of the motion which was occurring a little while earlier. It is true that no motion takes place in a single instant of time; but clearly whatever is in motion is determined, at each of the instants which can be specified as long as the motion lasts, to continue moving in a given direction along a straight line, and never in a curved one … (AT VIIIA 63-64; CSM I 242)

This is said in justification of the Second Law of Nature. This corresponds to the Third Law in *Le Monde*:

> that when a body is moving, even though its motion for the most part takes place along a curved path and, as we said above, it can never make any movement which is not in some way circular, yet each of its parts individually tends always to continue moving along a straight line. And so the action of these parts—i.e. the tendency they have to move—is different from their motion. (AT XI 43-44; CSM I 96).

In justification Descartes says that

> This rule is based on the same foundation as the other two: it depends on God's conserving each thing by a continuous action, and consequently on his conserving it not as it may have been some time earlier, but precisely as it is at the very instant that he conserves it. … (AT XI 44; CSM I 96).

In these passages Descartes is very explicit that God's conserving action occurs at every durationless instant, not moment, of a continuous motion. Moreover it is on this fact that he based his conception of the instantaneous activity of bodies as consisting not in a motion but in a *conatus* or instantaneous tendency to move, and the idea of reducing forces to such *conatûs* or tendencies to motion at an instant is absolutely crucial to the success of Descartes' physics. But the measure of force is quantity of motion, the product of bulk (*mola*) and speed (*velocitas*). Descartes asks us to imagine that God "placed a certain quantity of motion in all matter in general at the first instant he created it" (AT XI 43; CSM I 96), and then conserves this at every subsequent instant. Yet if God conserves a force of measure *mv* in a moving body at each instant of its continuous duration, then in any finite time corresponding to an actual motion, the measure of its quantity of motion will be (in modern terms) the time integral of this.

This seems to be the nub of Leibniz's criticism of the Cartesians in his *Specimen Dynamicum,* although the specifics of his argumentation are notoriously difficult to follow. There he argues that the product of mass (*mola*) and velocity (what he calls *impetus*) "is the quantity which the Cartesians usually call the quantity of motion, that is, the quantity of motion in a moment—though, to speak more accurately, the quantity of motion actually exists over time, and is the sum of the products of the different impetuses existing in the moving thing at different times and the corresponding time intervals" (Leibniz 1998, 157). That is, if the impetus is *mv*, then the quantity corresponding to an actual motion will be the integral of this over time; whereas if "we distinguish the present or instantaneous element of motion from the motion itself taken as extended over time," what the Cartesians usually call quantity of motion might more accurately be called "instantaneous quantity of motion [*quantitas motionis*]" (157).

Things are otherwise in a discretist physics, where the quantity of motion *mv* is specific to a given motion taking place over a small but finite time Δt_0. Any change in the quantity of motion Δmv produced by a discrete impact at the end of such a period will produce a new quantity of motion $mv + \Delta mv_1$ during the next time Δt_1 and so on. The motion will always be over a finite time, and the quantity of motion in such an interval will always have the dimensions of [*mv*], during which time it can correspond to God's action in so conserving it.

Yet in the passages quoted above, it is clear that God's conserving force is a *conatus*, a tendency to move at each individual instant. This should then correspond to what Leibniz called an element of a continuous motion, i.e. to *mdv*, not *mv*.[22] But Descartes rejected indivisibles in mathematics, and also the composition of a motion from such infinitesimal elements of motion.

What I want to suggest in the remainder of this paper is that this incompatibility between force as quantity of motion and force as instantaneous tendency to motion, and between moment as an interval of duration and moment as an instant, is not a difficulty that can be resolved by finding the correct interpretation, but a profound difficulty at the heart of Cartesian natural philosophy. Further, it was a difficulty right from the beginning of Descartes' efforts in mathematical physics, and an analysis of these beginnings can throw light on what that difficulty is and how it arose.

2. BEECKMAN'S DISCRETE MOMENTS

The first encounter we have with *moments* in Descartes' philosophy is at the very beginning, in confronting the first of the three challenge problems set him by his new friend and fellow mathematico-physicist, Isaac Beeckman, in the low countries in 1618. The problem Beeckman had set "Mr. René du Peron" was to determine "how much space a thing would cover by falling for a single hour when it is known how much it would cover in two hours, according to my foundations, *viz.*, *what is once moved in a vacuum moves always*, and supposing that there is a vacuum between the earth and the falling stone." (JIB I, 263; Nov 23rd–Dec. 26th 1618). Here the "thing" is a heavy body falling under the pull of gravity in a vacuum; as we shall see, Beeckman conceives gravity as acting by small, discrete tugs of corporeal spirits, pulling the heavy body "through mathematically divisible minimum physical space" (JIB I, 264). The way the problem was to be handled was by conceiving the motion moment by moment, as is made clear in Beeckman's formulation of the problem in his *Journal*:

[22] This has already been observed by Ed Slowik in his perceptive analysis of Cartesian physics: "In one sense, of course, [quantity of motion] is non-instantaneous; i.e., it includes the Cartesian concept of speed, which is only manifest over a non-instantaneous temporal period… Yet, [quantity of motion] is also closely tied to the instantaneous property which we can loosely entitle, 'tendency' …" (Slowik 2002, 113).

> In the first moment, as much space is covered as can occur as a result of the traction of the Earth. In the second, with [the stone] persevering in this motion, a new motion of traction is added, in such a way that twice the space is traversed in the second moment. (JIB I, 261; AT X, 58).

That is, the heavy body, on receiving a discrete tug, acquires a certain uniform motion, during which it covers a certain space. At the beginning of the second moment it receives a second tug equal to the first: this would take it through a space equal to the first. But in addition it continues with the same motion it had in the first moment (in accord with Beeckman's conservation principle), which would also take it through a space equal to the one it covered in the first moment. Thus in the second moment it has twice the force of motion, and so covers twice the space it did in the first moment. So as a result of these discrete tugs occurring at the beginning of each successive equal moment or physical indivisible of time, the speeds will be as 1, 2, 3, 4, etc in the successive moments; and given the equality of moments, the distances will therefore also be as 1, 2, 3, 4, etc in the successive moments, i.e. in arithmetical progression.

On this scheme, then, the motion is divided into individual moments, each moment of the motion being through so many moments or minima of space in each moment of time. Beeckman's own solution now proceeds as follows. If each hour were divided into 4 moments, the speeds would be as 1, 2, 3, 4, and 5, 6, 7, 8, with proportional distances covered in each moment. So after one hour the total distance covered would be as $1 + 2 + 3 + 4 = 10$, and after 2 hours, $10 + 5 + 6 + 7 + 8 = 36$, giving the ratio of the space covered in one hour to that covered in two as 10:36. If instead each hour is divided into 8 moments, the distance in the second hour will be as $9 + 10 + ... + 16 = 100$, giving a ratio of 36:136:

> So the latter eight terms to 16 are as 36 to 136, which is not yet as 1 to 4. If therefore the fall of the stone occurs through distinct intervals, with the Earth pulling by means of corporeal spirits, these intervals or moments will be so small that their arithmetic proportion, on account of the multiplicity of particles, will not be sensibly less than 1 to 4. (JIB I, 263; Nov 23rd–Dec. 26th 1618)

One can, in fact, derive a general formula. If n is the number of moments into which each hour is divided, the ratio between the distance D_1 covered in the first hour to the distance D_2 covered in two hours will be

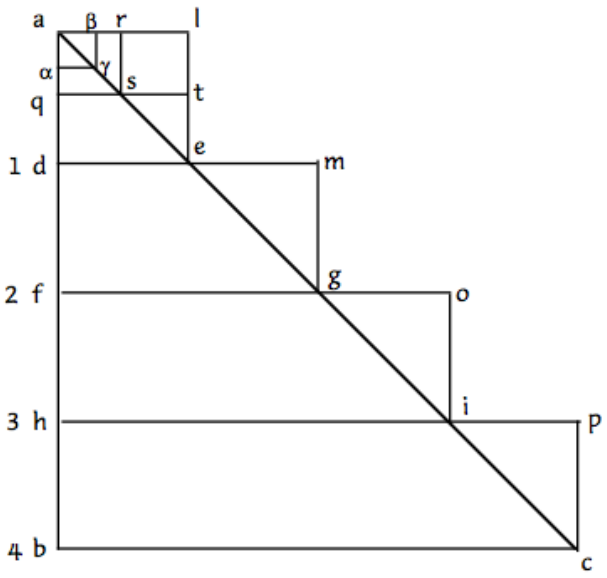
$$D_1:D_2 = {}^1/_2\, n/(n+1) : n\,(2n+1) = (n+1) : 2(2n+1)$$

$$= (1 + {}^1/_n) : (4 + {}^2/_n)$$

From this formula we can calculate that if there is one gravitational tug per minute, the ratio between the distance travelled in 1 hour to that travelled in 2 will be 61:242. If there are enough tugs so as to be insensibly small—say, 100 per second—the ratio will be 1: 3.9999944 —as Beeckman says, "not … sensibly less than 1 to 4".

How does this compare with Descartes' solution? Like Beeckman, Descartes begins by splitting the motion into constitutive moments, during which the body is conceived as moving with successively increasing uniform speeds. This is, in fact, how previous authors had proceeded, among them Leonardo da Vinci and Michel Varron. On Beeckman's scheme, these increases in speed are caused by the successive tugs of gravity. Descartes interprets them as successive attractive forces acting on the body, each causing an increase in the motive force of the body, whose effect is a proportional increase in speed:

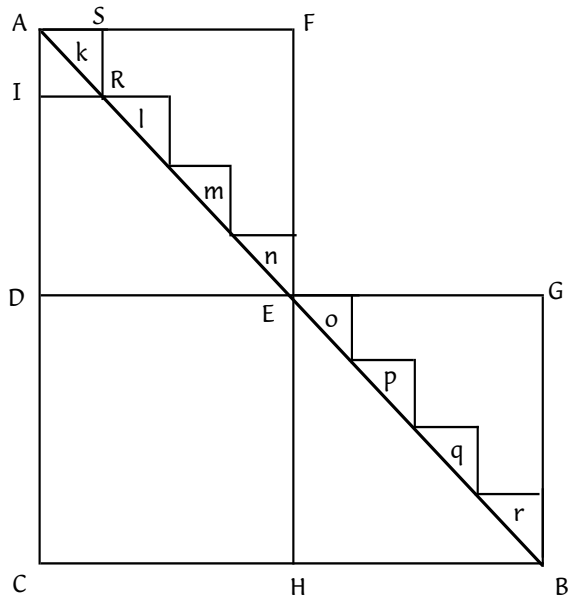
> In the problem proposed, in which it is imagined that at each time a new force is added by which the heavy body moves downwards, I say that this force is increased in the same way as are the transverse lines *de, fg, hi*, & an infinity of other transverse lines that can be imagined between them. To demonstrate this, I shall assume for the first minimum or point of motion, which is caused by the first attractive force of the earth that can be imagined, the square *alde*. For the second minimum of motion, we will have double this, namely *dmgf*: for the force which was in the first minimum persists, & another new one accrues equal to this. Likewise in the third minimum of motion... (JIB IV, 49; AT X 75)



Descartes' solution, however, is different from what Beeckman expected. Instead of interpreting the moments as physical minima of the motion, Descartes assumes, as Beeckman reports in his *Journal*, that the moments of the motion "ought to be imagined as indivisible & as containing no parts" (JIB IV, 50). To quote from Descartes' manuscript:

> Moreover, if for the minimum I assume a smaller one, such as $a\alpha$, the projecting parts will be still smaller, as $a\beta\gamma$, etc. And if, finally, I take for the minimum a true minimum, namely a point, then all the projecting parts will be nothing, since they cannot be the whole point, clearly, but only its half; but half a point is nothing. (AT X, 75 ff.)

In Beeckman's own words in his recounting of Descartes' solution, "the quantity of these increments [of space] will be null when the quantity of the moment is set as null. But such is the moment of space through which the thing falls." (JIB I, 262). Consequently Descartes effectively takes the limit of the total area as the number of moments—and therefore the protruding triangular spaces (*klmnopqr* in Beeckman's diagram (JIB I, 262) given below)—increase to infinity and as their breadth or quantity goes to zero.



We will come back to a consideration of Descartes' proof later. But leaving the mathematics aside, it makes a lot of difference to the physics whether the moments are physical, each being the duration of a uniform but very small inertial motion, or are mathematical fictions. Beeckman had clearly supposed the former, so that the minimum has some quantity. Indeed, on that supposition it is not actually possible to solve the problem completely, since one does not know this quantity, namely, how big the moments are. The problem needs *two cases* to be amenable to solution: say, "given that a body, falling a certain a distance in one hour, falls 51/202 of that distance in half an hour, how far will it fall in a single hour when it is known how far it would fall in two hours … ?"

Beeckman is aware of this, for he says:

> If, however, the moment or minimum of space has some quantity, there will be an arithmetic progression. Nor could it be known from one case how much [the falling stone] will cover in each hour; but there need to be two cases, in order for us to know from this the quantity of the first moment. Or so I had supposed; but since the supposition of an indivisible moment is more acceptable, I will not explain this in greater detail. (JIB I, 263)

In my example, the 51/202 ratio of distances covered in half an hour and one hour yields n = 100, so that the solution to the proposed problem will be: if the distance it falls in two hours is

D_2, then the distance D_1 it falls in one hour is (n + 1)/2(2n + 1) x D_2 with n = 100, i.e. D_1 = (101/402) D_2. In other words, the challenge problem Beeckman had set Descartes was in a sense a trick problem: the solution he expected was for Descartes to tell him "This is not solvable unless you tell me the size of the indivisible moments."

What then does Beeckman say of the solution he received from Descartes? He accepts it as an ideal limit of the case where *n* is finite but very large, since as *n* increases to infinity the ratio $(1 + {}^{1}/_{n}) : (4 + {}^{2}/_{n})$ becomes 1 : 4. But, he insists, the force of gravity is "not really continuous", as Descartes' solution presupposes, "but discrete":

> But we can at any rate accept this triangular proportion, not because there will not really be some mathematically divisible minimum physical space through which the minimum physical attractive force moves the thing (for this force is not really continuous, but discrete, and, as I would say in Flemish, "sy trect met cleyne hurtkens [it pulls by little jerks]," and therefore consists in the aforesaid increments, in a true arithmetic progression); but we can accept it, I say, because this <minimum> is so small and insensible that, because of the multitude of terms in the progression, the proportion of numbers does not sensibly differ from the continuous triangular proportion. (JIB I, 264)

There is thus absolutely nothing wrong with Beeckman's discretist mathematics, and contrary to the claims made in the literature, he completely understands Descartes' solution.[23] His reservations concern the compatibility of the mathematics with the underlying physical assumption of the discreteness of the force of gravity—compatibly with the mechanists' insistence on action by contact—and the consequent discretist interpretation of acceleration.

But what, in turn, does Descartes make of Beeckman's discretist approach? The answer is that he is disdainful and condescending. This can be seen by looking at a later episode, where Beeckman gives a similar discretist analysis to show that a body falling in a resisting medium such as air will finally reach a limiting or terminal velocity. In 1629 he reasons that since the resistance to a body falling in air will increase the faster it falls, there will come a point where

[23] Alexandre Koyré, for example, in his masterful work *Galileo Studies*, writes: "However, it should not be forgotten that Beeckman, while he was certainly a good physicist, was a rather mediocre mathematician." (Koyré 1978, 86).

the force of resistance equals the tug of gravity: “the attractive force of the Earth will not be able to be add anything to the motion of the thing, because the heaviness [*gravitas*] of the body into which it is going, that is, the air, is equal to the heaviness of the thing.” Beeckman discusses this with Mersenne, who asks Descartes for his opinion. Descartes shows that he understands Beeckman’s argument perfectly, for he gives the following account of it:

> The force that creates speed always increases uniformly (that is, by one unit at each moment), while the air resistance always impedes it in a non-uniform way (that is, less than a unit at the first moment and a little more at the second moment, and so on). Necessarily therefore, he says, there comes a point where this resistance is equal to the thrust which is due to the heaviness, when it reduces the speed at the same rate as the force of heaviness increases it. At the moment this happens, he says, it is certain that the weight does not fall more quickly than it did at the immediately preceding moment, and at the subsequent moments the speed will neither increase nor diminish, because after that the air resistance remains uniform … . (AT I, 91)

But having described the argument as “highly plausible”, Descartes then writes scathingly that “those who are ignorant of arithmetic might be convinced by it; but one needs only to be able to count to see that it is unsound.” (JIB IV, 172) In order to establish this, Descartes uses the technique of a continuous proportion between the speed and the resistance of the air to argue that an equality between force of motion and resistance can never be achieved, although it could perhaps be approached arbitrarily closely. Suppose, then, that a body falling in air that has a speed of 1 in the first moment has its speed reduced to 1/2 by air resistance during that moment. It follows that, receiving another increment of speed 1 at the beginning of the second moment to give it a speed of 3/2, in the second moment this speed will be reduced proportionately by the same factor of 1/2 to 3/4; its speed in subsequent moments will then be reduced by the air resistance to “$^{7}/_{8}$, $^{15}/_{16}$, $^{31}/_{32}$, $^{63}/_{64}$, $^{127}/_{128}$, $^{255}/_{256}$ and so on to infinity... Thus the reduction in speed due to air resistance is never as great as the increase in speed due to heaviness, which is one unit at every moment”. (172)

As a matter of fact, and for all his confidence in his mathematics, Descartes is wrong about this, for a body falling in air will eventually reach a terminal velocity where the resistance to its downward motion does indeed balance its weight, as Beeckman had argued. Indeed, this fact is

assumed by Galileo in his discussion of falling bodies in *Two New Sciences*: "a falling body acquires a definite speed, fixed by nature, a velocity which cannot be increased or diminished except by the use of force [*violenza*] or resistance." (Galilei 1954, 62-63) As Galileo points out, this "terminal velocity" depends on the specific gravity of the body as well as air resistance. The precise modern version of this fact is now known as Stokes' Law, derived by George Stokes in 1851. According to this law, a small body falling in a viscous fluid under its own weight will reach a terminal velocity when the frictional force combined with the buoyant force is exactly balanced by the gravitational force. For a spherical body of radius R and density ρ_p falling in a continuous viscous fluid of density ρ_f and viscosity μ, and with g the acceleration due to gravity, the terminal velocity will be

$$v_t = (2gR^2/9\mu)(\rho_p - \rho_f)$$

Since the specific gravity of such a body will be $S_{pf} = \rho_p/\rho_f$, this can also be written

$$v_t = 2gR^2\rho_f(S_{pf} - 1)/9\mu,$$

showing the dependence of the terminal velocity on specific gravity as posited by Galileo, and implicitly by Beeckman.

So what is wrong with Descartes' reasoning? The problem is the mismatch between his treatment of frictional force as acting continuously through each moment and of gravity as acting discretely at the beginning of each moment. He has used continuous proportions to treat the continuous reduction of speed due to air resistance (and we will identify difficulties with his use of continuous proportions below). On the other hand, though, he has followed Beeckman here in treating the force of gravity as acting discretely. This mismatch is masked by his considering the motion as though it were divided into moments.

On the other hand, despite Descartes' disdain, Beeckman's discretist reasoning is unexceptionable. If the gravitational force does indeed act by discrete impacts, one in each moment, each giving the body an equal finite increase in its quantity of motion; and if increasing air resistance is caused by the growing number of air particles the falling body collides with in successive moments, each collision causing it to lose a certain finite quantity of motion; there will come a discrete moment in which the quantity of motion accrued under

gravity will be equal to that lost in collisions with the air particles, and from then on the two will be equal. This reasoning, moreover, would successfully go over to the continuous case in the limit as the number of gravitational jerks increases to infinity and the size of the moments is shrunk to zero.

Effectively taking a limit is, of course, what Descartes had done in his calculation for Beeckman's original challenge problem. Note, though, that there is a real tension between Descartes' assumption there that the acceleration in fall is continuous and Beeckman's idea that gravity acts by discrete tugs—perhaps (as Beeckman explains elsewhere) by the actions of little screw-shaped particles coursing through the air or vacuum. This is significant in that Descartes himself believed that all action (including gravitation) is by contact. Moreover, as Klaas van Berkel has shown, Descartes' own explanation of magnetic force (if not gravity) as working by the action of little screw-shaped particles is one of the points on which he was deeply indebted to Beeckman's micro-corpuscular explanations (Van Berkel 2000). Finally, in his comments to Mersenne on Galileo's *Two New Sciences*, Descartes claims that although he once believed Galileo's "assumption that the speed at which a weight falls always increases uniformly, … I now think I have a demonstrative proof that this is not the case" (to Mersenne, AT II 386; CSMK 126). He also doubts that a falling body goes through all possible degrees of speed (AT II 399; CSMK 128).[24]

So Descartes certainly did not prefer his continuist solution to Beeckman's because he thought it was physically superior. On the contrary: its very physical untenability is most likely the reason he never published it.

3. Descartes' Continuist Mathematical Physics

[24] Indeed, according to the fine analysis of Carla Palmerino, Descartes was forced, because of the principle of the conservation of the quantity of motion, "to assume that all heavy bodies received a determined degree of speed at the very moment at which they collided with the subtle matter and that this speed increased with each successive collision, the limit of acceleration being given by the very speed of the pushing medium. The motion of fall was therefore not uniform. Instead, one had to assume that the higher the speed of the falling body, the less the effect exercized on it by the little pushes of the subtle matter." (Palmerino 1999, 294-95) This, of course, only heightens the irony of Descartes' criticisms of Beeckman. (My thanks to Geoff Gorham for bringing this article to my attention.)

What, then, explains Descartes' clearly articulated feelings of superiority with respect to Beeckman's mathematics? I cannot give a full answer to this question in the space I have allotted here. But an adequate answer would note the foundational importance to Descartes' natural philosophy of the idea of *instantaneous activity*. For Descartes, light is an instantaneous action, and there are no forces in things save for their instantaneous tendencies toward motion or endeavours (*conatûs*). On these insights depend Descartes' inspirational account of centrifugal force in terms of a composition of *conatus*, and his proofs of the laws of reflection and refraction, and much of his physics as well as his optics. (It will be remembered that the alternate title of *Le Monde* was *A Treatise on Light*.) A full account might well connect these insights with Descartes' mystical revelations in his stove-heated room, one of which was his notion that "instantaneous activity signifies creation";[25] which one might interpret as follows: creation is equivalent to God's continued working in the world, the signature of which lies in what happens at an instant. Thus there are no forces in things save for their instantaneous tendencies toward motion, and these are manifestations at each instant of the same divine force and action as was needed for the initial act of Creation: as we saw above, Descartes justifies his Third Law in the *Monde* by talking of God's conserving any thing by a continuous action "precisely as it is at the very instant that he conserves it" (AT XI 44-46; CSM I 96), and in the *Principles* Descartes claims that the equivalent Second Law is proved from "the immutability and simplicity of the operation by which God conserves motion in matter" (AT VIIIA 63; CSM I 242), an operation "by means of which the world is continuously conserved through an action identical with its original act of creation" (AT VIIIA 65; CSM I 243).

I call this fundamental feature of Descartes' natural philosophy his *instantaneism*. Schuster and Gaukroger have traced its beginnings in his physics to his solutions to the challenge problems in hydrostatics set him by Isaac Beeckman in 1618, where he conceives "weight as virtual motion, or more precisely instantaneous tendency to motion, which will play an important role in *Le Monde*" (Gaukroger 1995, 89; cf. Schuster 1977, I, 99). But they see this instantaneism as entering his metaphysics only much later, as part of a project of theological

[25] "The things which are perceivable by the senses are helpful in enabling us to conceive Olympian matters. The wind signifies spirit; movement with the passage of time signifies life; light signifies knowledge; heat signifies love; and instantaneous activity signifies creation." (AT X 218; CSM I 5)

apologetics. In contrast, I see a foundational connection between Descartes' natural philosophy and his metaphysics and theology *from 1619 onwards*: the reduction of forces in created things to instantaneous tendencies to motion, supported by God's continuous action, makes for a more profound dependence of created things on divine action than in Aristotelian and Augustinian philosophy, where things have natures and seminal reasons out of which their actions unfold. For Descartes, on the other hand, created things stand in need of the same divine force and action at every instant they endure, explaining the equivalence of conservation in existence with continuous creation.[26]

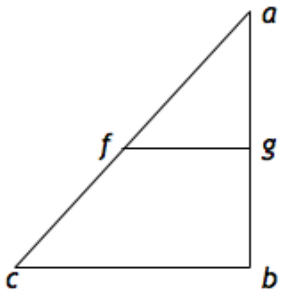
There is, of course, nothing like this instantaneism in Beeckman. And there is no doubt as to Descartes' pride in his reductive analysis of force to an instantaneous tendency or conatus, and the results he achieved on the basis of this conceptual advance. But this instantaneism goes along with a geometric mathematical physics: as we saw above, the straight-line motion that is natural to bodies is the instantaneous tendency to move along the tangent to a continuously curved trajectory. It is irrelevant that the real path will not be an exact curve. The very nature of bodies is found in their constitution in an instant, not in their constitution as bodies existing for a certain duration with the aid of God's conserving action.

I conjecture that Descartes' solution to Beeckman's 1618 challenge problem concerning free fall played its part in his reaching these new insights in 1619. That he was able to solve the problem without appealing to Beeckman's discrete "physical moments"—or, rather, by emptying them of all duration—may well have contributed to his liberation from atomism. (In the third challenge problem of 1618, he still appealed to Beeckmanian atoms, but they make no further appearance in his work after 1619.) But he was certainly pleased with the mathematical solution he achieved there, as is evidenced by his repeating a version of it to Mersenne eleven years later, in November 1629, and again in August 1634. In these later versions, though, Descartes no longer "takes the limit", but rather simply summarizes the triangular relation he

[26] Thus while I am in complete accord with Gaukroger that "Descartes is committed to the doctrine of instantaneous action from early 1619 onwards" (1995, 13), I reject his claims that "the metaphysical doctrine of the instantaneous nature of divine action first appears in 1640", and that it is "far more likely to be a metaphysical legitimation of a long-held physical theory" (13). As we have seen, it is in *Le Monde* as well as the *Olympica*, and appears to have the same metaphysical import from the beginning.

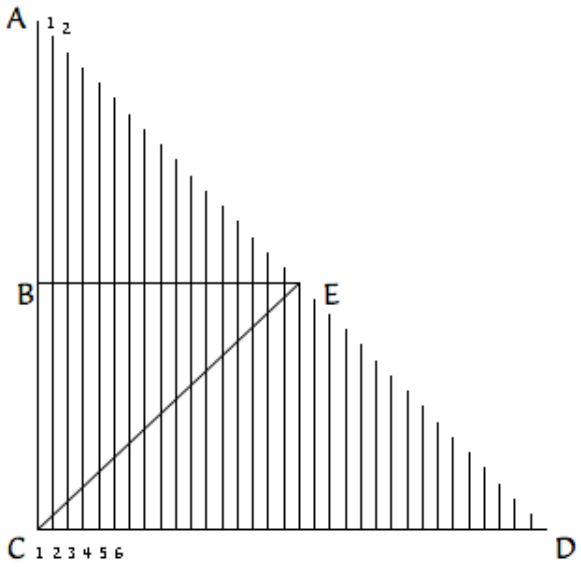
had established earlier. That is, he repeats the summary he had already given directly after the proof back in 1618, which went as follows:

> From which it clearly follows that if we imagine, for example, that a stone is pulled from *a* to *b* in a vacuum by the earth, by a force which always flows uniformly from it, with the prior force persisting, then the first motion at *a* is to the last at *b* as the point *a* is to the line *bc*; in fact the half *gb* will be traversed by the stone three times as fast as the other half *ag*, since it is pulled by the earth with a force three times as great: for the space *fgbc* is triple the space *afg*, as is easily proved; & one can treat the other parts of the line *ab* proportionately. (JIB IV, 49 ff.; AT X 75 ff.)



In this summary there is a noteworthy transposition from talking about temporal parts of the motion to spatial parts. In Beeckman's original diagram, and in the diagram accompanying Descartes' proof, the areas represented the motions, and also the spaces covered. This was reasonable, since in equal times the spaces would be as the speeds, where the speeds are assumed constant within each moment, according to Beeckman's discretist scheme. But the vertical line *ab* (*AD* in Beeckman's rendering) represented the time elapsed, while the horizontal lines would be proportional to the increasing speeds. In the above summary and diagram in the 1618 paper, however, the vertical line *ab* now represents the spatial distance through which the body falls. Similarly, in Descartes' letter to Mersenne in 1629 the line ABC is interpreted as the trajectory of the body's descent:

> Thus we get the triangle ACD, which represents the increase in the velocity of the weight as it falls from A to C, the triangle ABE representing the increase in the velocity over the first half of the distance covered, and the trapezium BCDE representing the increase in the velocity over the second half of the distance covered, namely BC. Since the trapezium is three times the size of triangle ABE (as is obvious), it follows that the velocity of the weight as it falls from B to C is three times as great as what it was from A to B. (AT X 72-73; CSMK 9)



This transposition from time to space, of course, is a mistake, as modern commentators have not failed to point out. Since the overall motion is not uniform, having the two *temporal* halves of the motion completed in times proportional to the areas is not the same thing as having the two *spatial* halves of the motion completed in times proportional to the areas. But it is important to ask how Descartes—a more than competent mathematician—could be led into making such an error. I have given a detailed analysis of this elsewhere (Arthur 2010), comparing Descartes' early efforts and also his later explication of the problem of fall to Mersenne. The answer, I believe, is this. Descartes is using a notion of the *total velocity* of the motion (*velocitas*), the swiftness with which the whole motion is accomplished. A motion, being the traversal of a certain space in a certain time, necessarily takes place over time, and how swiftly it does this is its velocity. This, of course, is the Aristotelian conception of *velocitas*:

> If one thing is swifter than another, it will cover a greater distance in an equal amount of time, and it will take less time to traverse an equal distance, and it will take less time to traverse a greater distance. Some people take these properties to define "swifter". (Aristotle, *Physics* vi.2, 232a 24-27)

In keeping with this, Descartes has applied the rule:

> *A body travelling with N times the velocity will traverse an equal space in 1/N of the time; or in an equal time will traverse a space N times as great*.[27]

This accounts for all the numerical examples Descartes gives in various places, and also accounts for why he is able to assume serenely that he can go back and forth between traversing intervals of space and traversing intervals of time. This is fine if we are considering a nonuniform motion as a whole, but not if we consider it part-wise, which is what Descartes is committed to by his application of the traditional Aristotelian notion of velocity and continuous proportions. You can see this from a specific example. Consider the motion through four equal spaces, say four cubits, in two halves. By the above hypothesis, the moving body covers these in times of ratios 1, 1/3, 1/5, 1/7, … to 1. But now if we calculate the overall times of fall through the first 2 and the second 2 cubits, these are as 4/3, and 12/35 s, resp. Thus the times of these two equal halves of the motion are in the ratio 4/3 : 12/35 = 1: 9/35, or slightly more than 4:1, whereas, as two halves of the motion, they should be in the ratio 3:1.

There is, therefore, a kind of serene overconfidence on Descartes' part concerning the adequacy of the mathematics of continuous proportions to handle continuously varying motions, and he does not notice how his odd number rule for the spaces covered differs from Galileo's rule for temporal parts of the descent. Neither author, of course, has the modern concept of instantaneous velocity; this must wait for Newton's development of his method of fluxions, and Leibniz's derivatives. They are both operating with something like the older Merton School concept of intensive quantity of the motion at each instant, what Galileo called the "degree of velocity". Galileo avoids Descartes' mistakes (mistakes that he himself had committed earlier) by taking time as the parameter of motion and using the mean speed theorem, which allows him to equate a uniformly accelerated motion with a uniform motion whose intensity is the same as the middle degree of motion. Real clarity about these matters was only achieved later: Le Tenneur (1649) dubbed the velocity of a uniformly accelerated

[27] That Descartes applied this rule is also noted by Palmerino: "By applying the medieval theory of proportions according to which 'for equal spaces velocities are inversely proportional to times elapsed,' Descartes arrives at the conclusion that the time required to cross the distance AB is three times longer than the time required for BC." (1999, 284)

motion whose intensity is the same as the middle degree of motion its "active velocity", and Gassendi (1646) emphasized the importance of taking time as the independent variable.

But our concern is with what is implicit in Descartes' analysis. In the 1618 paper he talks about the "first motion at *a*" and the "last motion at *bc*", even though there is, of course, no motion at an instant. As noted, what he means is more nearly the intensive quantity of motion. Thus "the first motion at *a* is to the last at *b* as the point *a* is to the line *bc*" expresses a proportion between the horizontal lines and the degrees of velocity or forces at each instant of the motion. In the 1629 letter to Mersenne, he does the same, although now the figure is transposed and it is the vertical lines that represent "the force of the speed impressed at each moment" by the combination of its impetus in the previous moment and the "additional impetus it acquires in each moment on account of its heaviness" (AT X 72; CSMK 8).

But here we are confronted with the problem noted above. In Descartes' physics the measure of the body's "impetus", the force of the body's motion, is quantity of motion, bulk times velocity. This derives from Beeckman's measure, "corporeity" times velocity (see Arthur 2007). Within one of Beeckman's discrete, physical moments, this is unproblematic: the force acquired by a body at the beginning of each moment is expressed in the increase in the quantity of motion in each successive discrete moment. Jean-Marie Beyssade contends that something similar is true for Descartes. Following Jean Laporte, he distinguishes a moment, a smallest but nonetheless divisible part of duration, from an instant, a limit of an interval or mere point of time. He then suggests that the *conatus "*accrued … in the minimal duration of a moment can be attributed to the moving thing taken at an instant" (Beyssade 1979, 141, n.5). One could also take this the other way around, and say that the divine action of conservation at each instant is manifested in the quantity of motion in the moment immediately following. But neither of these stratagems really solves our problem. God's action is supposed to take place at every *instant*, not through a moment, and to be manifested in the instantaneous activity of the body.[28] Light is a conatus, but this is an instantaneous tendency to move, not a very small

[28]Cf. Slowik 2002, 114: "In fact, bearing in mind that Cartesian motion does not take place at the level of instants, it becomes difficult to draw a sharp distinction between instantaneous tendencies towards

motion. The idea of capturing the force acting on a body at an instant is crucial to the success of Descartes' physics. But at an instant there can be neither action nor motion, nor duration.

4. Conclusion

Beeckman was not a time atomist. Although his moments are *individua*, they are still mathematically divisible. They divide the duration of a body being acted on by discrete impact-forces into discrete (very small) periods of duration (*momenta*) that are internally continuous.

Descartes' talk of moments of duration, and indeed his physics of action by contact seems to suggest a picture that is similar, where durations are divided into very small moments during which motion is inertial, and force and action are well defined. But this conflicts with the instantaneism of his natural philosophy, where continuous creation requires God's action at each single instant, and underpins the quasi-static reduction of forces to instantaneous tendencies to move. For the fact remains that for Descartes, at an instant, *a body has no duration, and consequently, strictly speaking, neither motion nor action*; so it is hard to see how instantaneous tendencies to move should follow the same laws as motions. And, if duration is made up of Beeckmanian moments, Descartes had no right to be so supercilious about Beeckman's discretist physics.

Dan Garber says in his discussion of Descartes on continuous creation and temporal atomism that this "was not an issue that worried him greatly. Nor should it have worried him, at least in connection with the foundations of his physics." (1992, 273) I hope that what I have shown here is enough to reverse that judgement. Whether or not Descartes "would have been somewhat surprised at the issues his commentators chose to emphasize" (Garber 1992, 273), both the *centrality of moments of motion and duration to his metaphysics*, and the *unsatisfactoriness of Descartes' treatment of them in his mathematical physics*, became a matter of great significance to his immediate posterity.

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motion, as measured by [the quantity of motion], and infinitesimal displacement, as incorporated in the [General Statical Principle]".

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